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ABSTRACT

This third volume of a review prepared for the Cockcroft Committee of Inquiry into the Teaching of Mathematics in Schools in Great Britain concerns research with relevance to the mathematics curriculum. The role of history is discussed briefly in the introduction, and has an impact throughout the report. The 13 chapters consider: (1) introduction, (2) early curriculum development and theories, (3) curriculum development in England in the early 20th century, (4) the Consultative Committee and mathematics, (5) later development of curriculum theories, (6) strands of curriculum development, (7) lessons to be learned, (8) evaluation in Britain, (9) evaluation in the United States, (10) the management of curriculum development, (11) centers and networks for curriculum development in mathematics, (12) common problems, and (13) the teacher's role in curriculum development. A bibliography is found at the end of the report. (MNS)

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Curriculum Development and Curriculum Research

A G Howson

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Mathematical Education

Prepared for the Committee of Inquiry into the
Teaching of Mathematics in Schools

Part C Curriculum Development and Curriculum Research: A Historical and Comparative View

A.G. Howson

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Contents

List of Abbreviations of Journal Titles

Full Title	Abbreviation
Acta Psychologica	Acta Psych
American Educational Research Journal	Am Ed Res J
American Psychologist	Am Psych
Arithmetic Teacher (USA)	Arith Teacher
Australian Journal of Education	Aust J Ed
British Journal of Educational Psychology	B J Ed Psych
British Journal of Psychology	B J Psych
British Journal of the Society for Clinical Psychology	B J Soc Clinical Psych
British Journal of Teacher Education	
Bulletin - Institute of Mathematics and its Applications	Bull IMA
California Journal of Educational Research (USA)	Calif J Ed Res
Canadian Journal of Behavioural Science	Canadian J Beh Sci
Canadian Psychological Review	Canadian Psych Rev
Child Development	Child Dev
Cognitive Psychology	Cog Psych
Cognitive Science	Cog Sci
Developmental Psychology	Dev Psych
Dissertation Abstracts International (USA)	Diss Abs Int
Education in Chemistry	Ed Chem
Education for Teaching	Ed for Teaching
Educational Research	Ed Res
Educational Studies	Ed Stud
Educational Studies in Mathematics	Ed Stud Math
Educational Technology	Ed Tech
European Journal of Science Education	Eur J Sc Ed
Human Development	Human Dev
International Journal of Mathematical Education in Science and Technology	Int J Math Ed Sci Tech
Investigations in Mathematical Education	Investigations in Math Ed

Journal of the British Association of Teachers of the Deaf	J Br Ass Teachers of the Deaf
Journal of Children's Mathematical Behaviour (USA)	J Child Math Beh
Journal of Curriculum Studies	J Curr Stud
Journal of Educational Psychology	J Ed Psych
Journal of Experimental Child Psychology	J Exp Child Psych
Journal of Experimental Psychology (USA)	J Exp Psych
Journal of Occupational Psychology	J Occup Psych
Journal of Psychology	J Psych
Journal for Research in Mathematical Education (USA)	J Res Math ED
Journal of Research in Science Teaching	J Res Sci Teaching
Journal of the Royal United Services Institute	J Roy Un Services Inst
Journal of Social Issues	J Soc Issues
Mathematical Gazette	Math Gazette
Mathematics in School	Maths in School
Mathematics Teacher (USA)	Maths Teacher
Mathematics Teaching (Br)	Maths Teaching
Monograph of the Society for Research in Child Development	Monog Soc Res Child Dev
National Elementary Principal	Nat El Principal
Physics Education	Phys Ed
Programmed Learning and Educational Technology	Programmed Learning and Ed Technol
Psychological Bulletin	Psych Bull
Psychological Monograph	Psych Monog
Psychological Review	Psych Rev
Quarterly Journal of Experimental Psychology	Quarterly J Exp Psych
Research in Education	Res Ed
Review of Educational Research	Rev Ed Res
School Science Review (Br)	Sch Rev
School Science and Mathematics (USA)	Sch Sci Math
Studies in Science Education	Studies in Science Ed
Times Educational Supplement	Times Ed Supp

Preface

This review is in three parts, differing to some extent in subject, in style and in format. In particular, in the section on the Social Context of Mathematics Teaching the main discussion of the material comes first and is followed by Conclusions and Recommendations. In the section on Learning and Teaching, the main conclusions to which the work leads are stated in the Introduction, in a similar way, the introductions to the chapters are used to place the material in perspective and to relate it to current practice.

Part C is essentially as it was presented to the Cockcroft Committee, but new references have been added where they are of particular interest and reflect more recent developments.

We acknowledge with gratitude the help of members of the Consultative Group: Mrs M. Brown, Mrs M.R. Eagle, Mr D.S. Fielker, Mrs A. Floyd, Dr G.B. Greer, Dr K. Hart, Dr A.G. Howson, Professor D.C. Johnson, Professor K. Lovell, Professor R.R. Skemp and Mr J.D. Williams.

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Chapter One
Introduction

A 1973 paper by D.F. Walker bears the title 'What curriculum research?'. It is a significant question, for, indeed, there are few research findings relating to the curriculum which can be quoted. Goodlad (1969) summed up the position when he spoke of 'the paucity of ordered "findings" from curriculum research - findings in the sense either of scientific conclusions from cumulative inquiry or of tested guidelines for curriculum decisions'. The position has not changed significantly in the years that have elapsed since the two papers quoted (and which referred to education in general, and not mathematics education in particular) were written.

The reasons are not hard to find. Although small-scale research into particular constituents of a curriculum can be readily carried out (see, for example, Janvier, 1978; Treilibs, 1979), investigations relating to the entire curriculum are much more expensive and difficult to mount. Such as have been mounted, and to which we shall refer later (e.g., NLSMA, IMU) have not been particularly successful in illuminating general issues. The construction of appropriate research models has not proved easy, for what was to be investigated was something exceedingly complex and the initial situation could often never be replicated.

Yet it would be foolish to ignore the developments of recent years simply because of the scarcity of 'objective' research findings. What the world has seen has been the greatest classroom experiment ever in the field of mathematics education. That experiment has taken a variety of forms and the outcomes have been many. Any attempt to summarize and to select must therefore reflect personal bias - and the reader must take this into account.

In a recent book, the American historian J.H. Hexter divided historians into two camps - the 'lumpers' who attempted to identify movements and themes, and the 'sifters' who always have a counter-example to hand to deny any general statement. The reader is warned that, on this occasion, the writer identifies himself with the 'lumpers', whilst recognizing that a 'sifter' might well tell a different story.

The reference to history is not merely a passing one, for if one is to understand the events of the past twenty years it is essential to see what preceded them. As Goodlad indicates, the best a curriculum developer can hope for is guidance and theory arising from cumulative inquiry and experience. It is essential then, when considering recent curriculum development and methods of dissemination and implementation, to have knowledge of the way in which such processes were traditionally carried out. Only when considered in a historical context can the achievements of the past twenty years be best evaluated.

Chapter Two
Early Curriculum Development and Theories

Curriculum development has, of course, a considerably longer history than 'educational research' as we now know it. The curriculum has gradually developed over the years and, whatever the institutional form which education has taken, there have never been long periods when curriculum developers have been inactive. Moreover, the need for curriculum development to be seen as a classroom activity rather than one to be carried out in committee is not recent. For example, Kant, in his essay, *On Pedagogy* (1803), writes that

> 'one fancies, indeed, that experiments in education would not be necessary; and that we might judge by understanding whether any plan would turn out well or ill. But this is a great mistake. Experience shows that often ... we get quite opposite results from what we had anticipated.'

Kant was also to draw attention to the lack of a theory of education. Attempts to remedy that deficiency have been made consistently during the last two centuries and, in particular, there have been several endeavours to provide a theory of the curriculum. Thus, for example, Humboldt's concept of 'Bildung' was to prove influential both in his native Prussia and outside. In particular, it was to influence Matthew Arnold and those two great nineteenth century parliamentary commissions on secondary education - the Clarendon and the Taunton. It is not relevant to our purposes to describe the curricular debates of over a century ago in detail. However, certain remarks made by Curtis and Boultwood (1965) when describing the various reform camps are significant. They refer to

> 'the majority of grammar school teachers who had not seriously thought about the issue (of curricular reform) and who through ignorance or complacency were prepared to continue with the restricted curriculum and the unsatisfactory methods of teaching which had been employed for several centuries. Unfortunately, this group formed the majority of secondary school teachers and it was to be many years before they were completely eliminated' (p.428).

Curriculum change, therefore, depended essentially upon the receptiveness and the ability and willingness to change of the bulk of teachers: it could not be imposed from on high even by those as eminent as Arnold or T.H. Huxley.

To complement Humboldt's theory of *Bildung,* educators could now also call upon psychological theories. Thomas Tate, an influential Victorian teacher-trainer, was thus able to invoke 'faculty psychology' to justify curricular decisions within mathematics education. Thus '*mental arithmetic* cultivates the memory and the powers of conception and reasoning. It also especially fosters the habit of promptitude, presence of mind, and mental activity' (Tate, 1857, pp.248-9), whilst 'algebra ... exercises in a way no other subject can, those powers of analysis and abstraction' (Tate, 1849, p.IV).

This nineteenth century view is still to be found in some quarters today (cf. the claims made on behalf of Euclid-style geometry). Thorndike (1903) summed up the doctrine as follows:

> 'The common view is that the words accuracy, quickness, discrimination, memory, observation, attention, concentration, judgment, reasoning, etc., stand for some real and elementary abilities which are the same no matter what material they work upon; that these elemental abilities are altered by special discipline to a large extent; that they retain those alterations when turned to other fields; that thus in a more or less mysterious way learning to do one thing well will make one do better things that in concrete appearance have absolutely no community with it'.

Thorndike, through his empirical work, was to put the final nails into the coffin of faculty psychology. Much earlier criticism had come from others, such as Herbart, and it was, indeed Herbartian psychology on which the mathematical reforms of the early years of this century were based.

> 'What is the bearing of all this on curriculum-building? The formalist frames his curriculum with the object of developing certain faculties. The Herbartian allows the mental content to grow by laying hold of ideas that will hook on to the old ... And the old will include not only old knowledge but old experience' (Godfrey and Siddons, 1931, p.15).

The author, writing in 1911, was Charles Godfrey, one of the leaders in the attempts to overthrow Euclid and to create a new secondary school mathematics curriculum (see, for example, Howson 1973, 1982).

We have here early evidence then of a willingness, indeed, desire, to plan mathematical curricula on educational and, in particular, psychological principles. Clearly, the need to plan curricula bearing in mind psychological theories is still essential. What our examples show is that these theories are not constant; they have evolved and will continue to do so.

Whether 'disciplinary values' should be abandoned entirely as a result of criticisms by Thorndike and others was now a major problem. What was it in mathematics education that had 'transfer value'? H.R. Hamley, himself a researcher in mathematics education, supplied an appendix to the Spens Report (1938) in which he quoted the words of the American Bagley taken from the 1923 Report of the (US) National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education*. Bagley expressed the view that transfer could take place if the teaching made the students conscious of procedures and of the value of general procedures: 'The theory of transfer through 'concepts of method' and 'ideals of procedure' furnishes a definite suggestion for teaching'. As we shall see, this was but one principle on which recent curricular activities have been based.

Chapter Three

An Early Experience of Curriculum Development — England in the early Twentieth Century

The early years of this century were marked by a frenzy of curricular activity in the country's public and grammar schools[1]. Godfrey, writing in 1920, compared pre-1903 to an ice age and saw in 1903 the break up of the ice. Not only geometry teaching was revolutionized, when Oxford and Cambridge revised their examination requirements so as to allow any proof 'which appears ... to form part of a systematic treatment of the subject', but, following the 1901 meeting of the British Association at which the current state of mathematics education in Britain was attacked many other changes were made. Thus, several schools established laboratories for practical mathematics (see, for example, Board of Education, 1912), schools adopted four-figure logarithm tables instead of the seven-figure tables previously used, the teaching of algebra was reconsidered, and trigonometry and calculus became established in the curriculum.

It was, above all, a time in which, following Perry's address in 1901, 'utility' was stressed. No longer was it thought appropriate for mathematics to be taught to an academic, and social, élite in a purely academic manner, divorced from its applications[2].

Even though these changes were to affect comparatively few schools and pupils (it has been estimated that just prior to the 1902 Education Act there were in England not more than 30,000 pupils in schools giving grammar school

1. See Price (1981) for a detailed account of these changes.

2. The period also saw the first attempt to impose a core curriculum on secondary schools (Board of Education, 1904): 'English Language and Literature, at least one language other than English, Geography, History, Mathematics, Science and Drawing, with due provision for Manual Work and Physical Exercise.' 'Not less than 7½ hours per week had to be devoted to Science and Mathematics, of which at least 3 must be for Science.' This detailed prescription of time-table hours was renounced in 1907.

education (Lester Smith, 1967, p.88)), they were not effected without difficulties and disappointments. Indeed, it could be argued that had developers in the 1960s had a greater awareness of these, they might not have fallen into the trap of undue optimism.

Since it is our intention to emphasize problems of curriculum development which would appear to have general currency, we briefly summarize particular examples and indicate typical references in the contemporary literature.

1. A great many schools 'went on teaching in the old way' and were relatively unaffected by change. The immediate effect in those schools which did change was 'probably rather chaotic' (Godfrey, 1920). Changes, then, are difficult to implement throughout a system and can cause considerable disruption until they are absorbed.

2. In many cases the intention of the reforms 'was quickly forgotten or not understood'. Teachers adhered 'slavishly to the text-book' and the 'unsatisfactory teacher' often showed a preference 'for the unsatisfactory text-book'. Absence of 'an accepted guiding principle' led to 'various fantastic and ill-balanced developments' (Palmer 1912, Godfrey 1912). It is easy, therefore, for reforms to become garbled and easier to transmit content via a text-book than general aims and principles. A lack of understanding of these ends can lead to a distortion of proposed means.

3 . Much depends on the quality of the teachers called upon to accept the changes. In the 1900s there were insufficient well-qualified teachers who understood the point of the changes - by being able to view them from above, from a position of strength. As Fletcher (1912) explained, 'the efficiency of individual teachers or even of a considerable body of teachers' cannot be measured by their academic qualifications, for lack of these can be compensated for by personal qualities. Nevertheless 'when the question is not of an individual or of a small group, but of a large number, it remains true that the lack of good qualifications must seriously limit the efficiency of teaching'.

4. In-service education (which was not well developed in an institutionally-based form in the 1900s) is not as easy to effect as might at first appear. 'It is not sufficient to take the ordinary teachers, whose educational experience is limited ... and by a short course of lectures arouse in them a temporary enthusiasm for new methods. I have myself tried that plan. The enthusiasm was there and all seemed intelligent and interested until I followed the teachers into the schools and heard the reproduction and extension of what I had taught. In many ways the patching of the new

cloth into the old garment was a lesson in more senses than one ... ' (Shaw 1901).

5. The examination system dominated the reforms and through its 'backwash' into schools helped distort aims and methods (Hobson, 1910, Godfrey, 1912).

One observes then that educators in the first two decades of this century faced similar problems to those which we do nowadays. In their case a period of frenzied activity[1] was followed by a period of retrenchment and consolidation: a period which witnessed much excellent mathematics teaching in our grammar schools. Yet much of value was lost in that period of retrenchment. Work on transformation geometry (see, for example, Dobbs, 1913) came to a halt, to be revived decades later, attempts to present mathematics as a single subject rather than as three or four separate branches (Mair, 1907) were temporarily set aside, and, with an accompanying sigh of relief (see, for example, Mayo, 1928, pp 129-146), schoolmasters closed their mathematical laboratories and gave up 'pandering to the spirit of mere utility in education'. Even more importantly, the need constantly to keep the curriculum under review was neglected: a new 'ice age' came about, to be followed by yet another frenzied bout of activity to mark the thaw.

We end this section with a review of those two decades by P.B. Ballard. Writing in the 1920s, Ballard compared the recent reforms (which also encompassed changes in the elementary schools) to the Bessemer process for making steel in which one first takes all the carbon out of the iron and then reintroduces 'just the right amount'. It is, as the reader will see, prophetic, for the battle between the idealist who emphasizes understanding and a broad outlook, and the practical man who calls for the restoration of techniques 'in the interests of efficiency and economy of learning' is still being fought.

'Ever since the first unhappy days of payment by results, mechanical arithmetic ... claimed a large amount of the teacher's attention. Not a day passed ... without the

1. We note that during the period 1900-1910 the membership of the Mathematical Association more than doubled; presumably reflecting the belief that the Association could provide assistance at a time of stress. No such expansion took place from 1962 onwards - when the Association adopted a neutral stance towards reform. In contrast, the membership of the Association of Teachers of Mathematics rose from 1570 in 1962 to 6025 in 1967. Since that time there have been no significant changes in the membership of either association.

children having to undergo a severe drill in computation. They had to produce their daily tale of correct sums. With the advent of happier days came doubts respecting the value of the facility gained by these laborious exercises. It was seen that accuracy was secured at the expense of intelligence. The children got their sums right, but they did not understand what they were doing; they could not apply the principles to the ordinary affairs of life. Then started an intelligence crusade. Mechanical arithmetic was dubbed stodgy and superfluous. Problems and practical arithmetic were proclaimed the only means of arriving at an intelligent grasp of the principles of numbers. The bulk of the schools adopted this faith and put it into practice. But it did not bring forth good works. The quality of the arithmetic rapidly declined. Even the problems themselves were badly done. The gist of the problem was too often obscured by a multitude of words, and its solution hampered at every point by sheer inability to compute. It thus came to pass that a small proportion of the rejected practice in sums and tables found its way back ...

(There was also) an attempt made to derationalise geometry. It was held that Euclid's reasoning was too difficult for young minds, and that youths and maidens in their early teens should be limited to geometry that was wholly practical. They should know the 'how' but should not trouble themselves about the 'why'. They should discover empirically geometric truths which they could later in their school career learn to prove by deductive reasoning. Even for this later stage Euclid's treatment was considered clumsy and out of date. As a consequence reasoning was left out in the early stages and Euclid left out in the later. There is now a tendency to restore a modicum of reasoning to the very first steps in the course. And if Euclid himself has gone for good, what he stands for is coming back ...

All these are examples ... of the Bessemer process at work in the schools. It is not, properly speaking, a case of the swing of the pendulum - a phrase with which we are wont to dismiss all cases of shift and change that we cannot account for ...

Has the Bessemer process benefited education or has it injured it? ... Every impartial witness will I think agree that it has [perceptibly pushed things forward]; that its influence has been distinctly on the side of good. It has been an experiment on a colossal scale. And it has enormously enhanced our knowledge of educational values - of the possibilities of certain branches of instruction and their limitations. It is true that the same knowledge might have been more easily gained by a series of careful experiments on a much more limited scale;

but by no means than by this large and universal trial could the verdict be readily brought home ... It has not remained mere theory; it has become theory embodied in practice. It will be noticed that the elimination has always been the elimination of drudgery or of premature reasoning; and the result has been the removal from the curriculum of indigestible material and the reduction of dull mechanical grind to the lowest limits compatible with efficiency of learning.

[The Bessemer process in education] is going on at the present day, and it will go on in the future ... Indeed, [it], or some similar mode of change and adjustment, is quite inevitable. It is inevitable because the dreamer and the practical man are always with us ... And the dreamer, the idealist, will always call for elimination, in the interests of intelligence and a broad outlook; and the practical man will always call for restoration, in the interests of efficiency and economy of learning ...' (Ballard 1925, pp. 223-236).

Chapter Four
The Consultative Committee and Mathematics

In retrospect the most interesting feature of the reports on secondary education issued by the Consultative Committee of the Board of Education (Hadow 1926, Spens 1938), the national Advisory Councils (Hamilton Fyfe 1947, Newsom 1963, Crowther 1959) and the Secondary School Examinations Council (Norwood 1943, Beloe 1960), and of the Board and Ministry of Education pamphlets (1935, 1958), is their unanimous view concerning the distribution of mathematical abilities and their acceptance of the desirability of, and necessity for, differentiation of students and curricula.[1] It must be stressed that no strong empirical evidence is advanced to support the views expressed; the same could be said, however, with respect to the opinion which gained ground in the late 1960s and early 1970s that such differentiation was neither desirable nor necessary.

In many ways this *volte face* in opinion and practice within the teaching profession is the most significant change to have occurred in mathematics education in England and Wales during the last fifty years. For that reason it is important to reconsider the arguments advanced in these official reports and pamphlets.

The Hadow Committee was asked to consider *inter alia*, the curriculum of those pupils who received education at schools, other than grammar schools, up to the age of 15. It was, then, concerned with a burgeoning sector of education, not all of which was included in the years of compulsory education. The report foreshadowed the split between primary and secondary education, the establishment of a leaving examination for the 'average' child, and the setting up of a tri-partite system of education. It was felt strongly that the vast majority of children required a secondary education having 'a less "academic" bias' and giving 'a larger place to various forms of practical work' than was provided in the grammar schools. This belief was exemplified in the report's comments on mathematics. There was dissatisfaction with the way in which arithmetic

1. Here, 'curriculum' is taken to encompass content (syllabus), method and assessment.

dominated the curriculum of primary and senior schools and a wider view of mathematics was called for: one which would provide children with the knowledge 'necessary for the intelligent comprehension of some of the problems of everyday life'. New teaching methods were required and so were curricula which were adapted to local needs and conditions. Yet although the committee was unanimous in agreeing on the broad aims of mathematical education, it is noticeable that even the framing of a specimen syllabus for a Modern school led to some dissension.

The Spens Committee was concerned with Grammar and Technical schools rather than Modern schools. Yet even it was concerned with the 'latent prejudice against technical or quasi-vocational studies' which it observed in schools. In general, its comments on the curriculum had that timeless air to be noticed in the Hadow report (and which one assumes will be present in reports still to be written):

> 'The curriculum should be thought of in terms of activity and experience rather than of knowledge to be acquired and facts to be stored. Both the conservative and creative elements ... must be represented in the curriculum ... The studies of schools ... should be brought into closer contact ... with the practical affairs of life ... The School Certificate Examination dominates the curriculum unduly. It should follow the curriculum, not determine it (Spens Report).

The Spens committee had some interesting recommendations to make on mathematics. 'The content of school mathematics should be reduced'. 'If it be taught [as one of the main lines which the creative spirit of man has followed in its development] it will no longer be necessary to devote the same number of hours to the subject'. The committee felt that all pupils should study science or mathematics to the age of 16, but that the latter could be dropped in the 'third and later' years of study. Mathematics teaching suffered 'from the tendency to stress secondary rather than primary aims' ... instead of giving 'broad views' it concentrated too much on tricky problem-solving. 'It is sometimes utilitarian even crudely so, but it ignores considerable truths in which actual mathematics subserves important activities and adventures of civilised man'. The type and 'rigour' of the logic it presented had 'not been properly adjusted to the natural growth of young minds'. 'These defects are largely due to an imperfect synthesis between the idea that some parts of mathematics are useful to the ordinary citizen or to certain widely followed vocations, and should therefore be taught to everybody, and the old idea that, when mathematics is not directly useful, it had indirect utility in strengthening the process of reasoning or in inducing a general accuracy of mind' (pp. 235-242).

The 'dreamers' or 'idealists' were, then, well represented on the Hadow and Spens Committees. It was the administrators whose views appeared to dominate the report of the Norwood Committee - a Committee of the Secondary Schools Examination Council, appointed in 1941 to consider 'curriculum and examinations in secondary [grammar] schools'. The Norwood Committee believed not only in the tripartite system of education but in the tripartite nature of 11-year-old children. Not only were distinguishing features easily recognized but it seemed 'that the Almighty had benevolently created [these] three types of child in just those proportions which would gratify educational administrators' (Curtis, 1952, p.144). The psychologists too objected: 'The proposed allocation of all children to different types of school at the early age of eleven cannot provide a sound psychological solution ... the grounds for [so doing] are administrative rather than psychological' (Burt, 1943, quoted in Rubinstein and Simon, 1973, p.30). Such reactions to Norwood might well have precipitated what Ballard would have seen as the first stage of the Bessemer Process on the differentiation of pupils and curricula.

Norwood considered the problem of 'mathematics for all' and felt that 'the majority of pupils' in grammar schools should study mathematics until 16+. However, it stressed that more differentiation within grammar schools was required. More of the able mathematicians should be encouraged to aim for 'Additional Mathematics' and, for others, a course less demanding than 'Elementary Mathematics' should be made available.

Unlike any other committee, Norwood was somewhat complacent about mathematics teaching:

> 'the various branches ... have coalesced, dead matter has been pruned away ... and laborious formal proof and rigidly logical sequence have been replaced by shorter methods and by the demonstration of mathematical principles and their practical application' (p.105).

This is a view of mathematics teaching which is not only at odds with personal memories, but with the report of the Jeffery Committee soon to follow that of Norwood.

Whereas the English and Welsh reports mentioned so far were concerned with special sectors of what we now term 'secondary education', it was on the whole range that the Advisory Council on Education in Scotland reported in 1947. The Hamilton Fyfe Report rejected a 'tripartite' system for Scotland, but nevertheless had some outspoken remarks to make on the need for the differentiation of curricula. In its 'omnibus' schools differences of ability had to be recognised and dealt with immediately. Even a first-year 'common course' was rejected as 'bound to involve the sacrifice either of the few to the many or the many to the few'. Mathematics teaching was denounced as 'being too formal' and 'divorced from practice': it needed an 'overhaul' In particular, there was a need for 'very simple courses' 'for D, E and the weak C' pupils. Indeed, the viability of teaching such children 'mathematics' other than 'some skill in calculation and some understanding of number and spatial relationships' was not only questioned, but ruled out.

> 'If there be any who still hold that, without [Mathematics and Foreign Languages], there is no secondary education, then they must face the distressing fact that nature has denied secondary education to a large part of normal humanity; for the evidence is conclusive that very many children, perhaps even a majority, are incapable of progressing any distance in these subjects or of extracting any substantial benefit from their study' (Hamilton Fyfe Report, p.20).

Quite what the 'evidence' which supported this conclusic was we do not know. Yet it is important to ask whether the experiences of the last thirty years have adequately served to rebut this statement.

As with the other reports there was the statutory paragraph(s) devoted to the problems of teaching mathematics to girls. Scottish opinion was that in the highest and lowest ability ranges no changes were necessary:

> 'Our evidence suggests, however, that at the middle levels there should be a separate course for girls, though some of our witnesses suspect that the disability of girls in this subject is conventionally assumed rather than real' (op. cit. p.99).

We note that amongst the casualties of recent reforms have been those 'Arithmetic for Girls' texts which flourished in the first half of this century.

In general, however, the reports mentioned so far, the pamphlets of the Board and Ministry of Education, and those Reports of the Mathematical Association written in the 1940s and 1950s which referred to secondary education are unanimous in recognizing varying levels of abilty and the benefits of setting (even within grammar schools; see, for example, Ministry of Education, Pamphlet Number 36, 1958, p.102). It was generally accepted that pupils could be roughly classified into three main groups: those who could cope with and derive benefit and even enjoyment from an academically-biased course in mathematics; those who derived little satisfaction from abstract mathematics but who were motivated by, and could deal successfully with, its more technical aspects, e.g., technical drawing; and those to whom the subject posed great difficulties and who, it was felt, responded best to an approach which was pre-eminently practical and clearly linked to other subjects of the curriculum.

This need for differentiation of curricula was also stressed in the Beloe Report on secondary school examinations other than the GCE: 'What is wanted is not an examination which simply reproduces the GCE pattern at a lower level, but an examination with a different character and aims ...' (Beloe Report, p.31).

The extent to which CSE with its increased opportunities for teacher involvement, failed to live up to its creators' expectations can be judged by comparing the regulations and syllabuses of any GCE with those of any CSE board.

Again, with the benefit of hindsight, one can only marvel at the lack of foresight shown by the Beloe committee who not only failed to see the extent of the difficulties which the institution of the CSE would cause, but also the way in which the British educational system would evolve so rapidly. Thus, for example, we have a description of the 1960s as 'a period when the modern school ought to be encouraged to grow steadily in stature ...' (op. cit., p.24).

Whether schools should limit entry to the new examinations - which the Welsh initially believed should be aimed at children within the 60th-80th percentiles - and the effect which permitting candidates to enter for both CSE and O-level would have on the character of the CSE examinations were problems which were either ignored or shrugged off.

Indeed, curricular matters were given scant consideration - there are, therefore, no specific references to mathematics and its particular problems.

Clearly, the hope of the Welsh contributors to Beloe that with the advent of the CSE 'children would have an opportunity of taking an examination designed to meet their own particular needs' has not come to pass - at best, the majority of the mathematics examinations would seem attuned to 'ability' rather than 'needs'.

The Newsom Committee which reported in 1963 on 'the education between the ages of 13 and 16 of pupils of average or less than average ability' was to provide the final impetus to the raising of the school-leaving age to 16. It was a report notable most of all for its sociological concern. So far as mathematics was concerned it emphasized the need to provide pupils of 'average and less than average ability' with a concrete and practical course: 'the abstract thinking of mathematics ... will be beyond their reach, but they can acquire a commonsense in figure handling ... It is important that these boys and girls should penetrate as far as they can by secondary ways of learning into the world of mathematics ... At present this is not very far ... ' (Newsom Committee Report, p.113).

In a sentence which is a masterpiece of qualification, Newsom suggested that 'Even the 'new mathematics' may have something to offer for some of our pupils where there is a well-qualified and well-versed enthusiast on the staff' (op. cit., p.149).

A more telling comment, however, concerned arithmetic:

> 'Many of our children will not have acquired facility in the four processes of addition, subtraction, multiplication and division in the primary schools, at least not with any permanence or security, and when, at the secondary stage, the four processes are found to have been inadequately learned, mere repetition of elementary exercises will fail to put matters right' (op. cit., p.148).

Exactly what will put matters right is still to be ascertained. Certainly, the measures suggested by Newsom will help ameliorate matters. The general extent to which the problem and the need for prompt remedial action on entry to secondary education has been recognized is, however, not clear. Neither is it obvious that this need can be adequately met in combined first-year courses (cf. the quotation from the Hamilton Fyfe Report).

The needs of older pupils were considered in *15-18*, the report of the Crowther Committee of the Central Advisory Council (1959). The Crowther Report introduced us to the concept of 'numeracy': 'the ability to reason quantitatively, but also some understanding of scientific method and some acquaintance with the achievement of science'. (A concept which we note has been narrowed in the intervening years.) Crowther hoped that steps would be taken to make all sixth-formers 'numerate': every educated person should be able to think quantitatively and have a rudimentary knowledge of statistics. This call was later to be reinforced by the Dainton Committee (1968). The reasons why little was done, or indeed could be done, is, however, evident in the statistics produced by Crowther which showed that few graduate mathematicians were at that time being recruited to modern schools and that no longer were the grammar schools attracting the good honours mathematicians. Crowther felt that a review of the sixth-form curriculum was needed: mathematics and science curricula were overloaded. (When the changes did come, the pressures were, in fact, increased rather than eased.) The report, then, contains expressions of discontent, but even allowing for the increased representation of industry on the Central Advisory Council, Crowther, like his predecessors, does not lay particular stress on the mathematical shortcomings of entrants to industry and commerce. There have always been, as we have shown, criticisms of mathematics education in schools, but these tended to come largely from within the system, from educators. The complaints made in the 1970s by industry would seem to be on a different scale to those which were made in years gone by. It would, however, be simplistic to regard this as evidence of falling standards; the increased demands of industry and the rapid growth of higher education which has channelled off many who would otherwise have entered industry from school must also be considered.

To date the emphasis has been on reports concerned with secondary education. In that sector enormous changes have occurred affecting school population, organization, and external examinations. The position has become more complicated and new constraints and problems, e.g., the reluctant 15-16 year old, have been created. The primary school has been spared these problems - indeed, one major constraint, the 11+ examination, has almost disappeared. This is not to say that there have been no changes in that sector, for the establishment of middle schools and, as a consequence, first schools, is but one example of structural change. Nevertheless, the changes in the primary school have been directed more at curricular ends and exhibit a greater sense of continuity. This is particularly noticeable in the field of primary mathematics.

The Hadow Report of 1931 devoted some pages to the consideration of 'arithmetic and simple geometry'. It stressed the fact that the fundamental processes of arithmetic 'should become automatic' before the child left primary school, but, notwithstanding this, felt that too much time in primary schools was devoted to arithmetic and insufficient to geometry.

The hoped-for widening of the primary mathematics curriculum was slow to take place. The Plowden Report, *Children and their Primary Schools* (1967), thought 'it is perhaps only in the last five or six [years] that the new ideas have spread so as to affect at least a majority of primary schools, and to justify the name of revolution in a substantial majority' (p.235). Highly influential in this movement were the Mathematical Association's 1956 Report, *The Teaching of Mathematics in Primary Schools*, the Schools Council's Curriculum Bulletin *Mathematics in Primary Schools* (1964), and (perhaps to an even greater extent) the meetings and workshops mounted by the authors of these books, and the propagation of their ideas in initial training courses.

Plowden was, of course, able to refer not only to the fact that 'rapid revolutions are not common in English education', but that a project had been established by the Schools Council and the Nuffield Foundation which would deliberately set out, if not to bring about a *coup d'état*, at least greatly to accelerate change. Indeed, Plowden was the first report of those we have described to recognize some of the practical problems of curriculum development - for, even as early as 1967, these were becoming apparent:

> 'this sort of approach demands a considerably greater knowledge of mathematics or rather degree of mathematical understanding in the teachers than the traditional one. If the children have to think harder, so do the teachers. Some have difficulty in identifying the mathematical aspects of topic work. Many ... faced the change with a poor equipment of mathematical training' (Plowden Report, p. 238).

In general, however, the Plowden Report reflected the optimism and hopes of individuals rather than facts (which, at that time, were hard to come by). Thus the question 'Will the modern methods lead to a decline in computation and accuracy?' was answered by 'Accuracy, indeed, is likely to improve, since in this kind of work there is a built-in incentive to accuracy' (p.238). An answer which was possibly correct - given a competent teacher who appreciated what was being attempted - but which did not take into account the difficulties of dissemination and implementation with the opportunities offered for garbling and misunderstanding. Nevertheless, the summing-up that

> 'the specially devised machinery of the Nuffield project must be succeeded by established machinery. The future will depend upon the extent to which we can produce teachers with the necessary knowledge and understanding to use and improve upon the material made available to them, and to keep themselves up to date' (op. cit., p.239)

cannot be faulted. Neither can the assumption that this will only come via 'permanent arrangements ..., locally and nationally for in-service training'.

Chapter Five
The Later Development of Curriculum Theories

In his booklet *Educational Theories* (1927), Sir John Adams makes no mention of any theories relating to the curriculum as such, although he does make a passing reference to Dewey's work in the USA and its sociological bias. The benefits of basing learning upon the environment, with the use of the 'project method', of 'learning by doing' and 'learning by experience' had, indeed, been advanced by Americans such as Dewey and Kilpatrick. It was a method which had considerable appeal to English educators: it was consistent with Adams' view that 'the teacher's function as educator is to make himslf dispensable'. What would not have appealed greatly to Adams, and which he neglected to mention, was the growth of a theory of the curriculum based on what we have come to term behavioural psychology.

Here the starting point was the introduction in industry of assembly-line work and job evaluation. Great economies and increased output followed careful 'job analysis'. Might not such techniques be applicable to education? These ideas were developed by Bobbitt in *The Curriculum* (1918) and *How to make a curriculum* (1924). Charters in 1923 laid down as his first three rules of curriculum construction:

> 'First, determine the major objectives of education ...
> Second, analyse these objectives into ideals and activities and continue the analysis to the level of working units.
> Third, arrange these in order of importance.'
> (Charters, 1923, pp.486-7).

Thorndike, as was mentioned above, had carried out empirical research to demonstrate what was *not* acquired when one carried out certain educational tasks. Not unnaturally, he also investigated exactly what outcomes could be expected in such circumstances, and this led in 1922 to his influential *Psychology of Arithmetic* in which he was to develop the instruments of behaviourist learning theory. Even earlier (1912), he had pointed the way to programmed learning by arguing that personal instruction for all would be guaranteed if only one could produce a book arranged so that page two was invisible until the directions outlined on page one were successfully carried out. A machine equivalent to such a book was, in fact, produced by S.L.Pressey in the 1920s.

This work was, however, not to have any immediate effect on American schools: for example, programmed learning was to remain dormant for almost three decades. Yet the orientation of Bobbitt and Thorndike was greatly to affect later American developments. Thus the specification of objectives in behavioural terms has been a feature of much US research and development work and several attempts have been made to provide mathematics programmes consonant with the theories of behavioural psychology. 'Teacher-free' programmes have never exercised much appeal in Britain where the curriculum has been traditionally teacher-controlled. Nevertheless, as we shall see in a later section, some British projects have attempted to absorb some of the lessons of the behaviourist school. Moreover, the coming of microprocessors will almost inevitably reopen the questions of the place of programmed (and/or computer-assisted) learning in schools.

Reference must also be made to the work of Ralph Tyler, and in other sections of this report attention is drawn to other Chicago-based activities: those of Bloom, for example.

America was to produce other notable workers in the field of curriculum theory, such as Hilda Taba and J.I. Goodlad. Yet when curriculum development began in earnest in the late 1950s, such theory could provide little practical assistance to developers (even if, as was not the case, the developers had been fully aware of the existence of a body of theory). Indeed, the effect of the developmental activities was to initiate a new trend in curriculum theory: the (partial) abandonment of attempts to provide a prescriptive theory, in favour of empirical studies of curriculum development in practice in the hope that from these would emerge a theory having a descriptive, explanatory function.

The English have, however, tended to be impatient of theory and have preferred a pragmatic approach. What happened when a collection of pragmatic Englishmen met Goodlad, Tyler and other theoretically-oriented North Americans head on is amusingly described in Maclure (1968):

> 'The effect of Goodlad's address was to polarize at the outset some of the differences between the English and the North Americans. It confirmed many of the English in their deeply-held suspicion of educational theory and their amazement that an apparently pragmatic matter like the

curriculum should be turned into a topic for academic abstractions. It did more than this. As a matter of sober reporting, it has to be recorded that it left many members hopping mad' (Maclure,1968, p.9).

Maclure remarks that many of the English present

'retreated into [the myth of the autonomy of the teacher]' whereas 'the Americans seemed to react by advancing still further into theory and abstraction, while the Canadians took refuge in the administration of curriculum renewal' (op. cit., p.10).

In the years since 1967, curriculum theory has become respectable in certain British circles; books are written and courses given. Yet the theory presented is usually one which embraces the curriculum as a whole, and the developer may well be justified in expecting that he will obtain little practical assistance from a theory which applies equally to mathematics, modern languages, home economics and modern dance.

The needs, then, are obvious; the solution is not.

Chapter Six
Strands of Curriculum Development

There is a tendency for all recent developments in mathematics to be lumped together under the general heading of 'modern maths' (or 'modern math' depending upon which side of the Atlantic the discussion is held). This is misleading, for it is clear that innovators were motivated in differing degrees by a number of educational and mathematical forces. It is useful then before we attempt to describe the various experiments and their aftermaths to try to identify different 'strands' or 'forces'. The classification given is that of Keitel (1976). It is explained in greater detail and illustrated with case studies in Howson, Keitel and Kilpatrick (1981).

Five 'strands' or 'movements' are identified. However, it must be stressed that the classification is somewhat arbitrary: certainly the labels cannot be attached to well-defined equivalence classes of either developers or projects. Life is not so simple or neat. Usually a number of influences can be discerned: what emerges is a mix with certain ingredients predominating.

Nevertheless, even such a rough classification can help us to appreciate the wide variety of objectives and approaches.

1. *New math:* this was basically a content-oriented movement which showed little concern for pedagogical matters. It was founded on the belief that existing syllabi were mathematically inappropriate and that new content would not only result in the attainment of new, and more relevant, objectives, but could also assist in the achievement of old goals. Thus, for example, emphasis on the commutative, associative and distributive laws would help to remove misunderstandings in number work and algebra and thus facilitate the acquisition of essential techniques. Moreover, 'concepts' such as sets, relations, matrices and functions would help to 'unify' mathematics.

This was the 'strand' which had most immediate impact. It was this movement which dominated the influential Royaumont (OEEC, 1961a) and Dubrovnik (OEEC, 1961b) seminars.

their backing, and its influence was manifested in such projects as the School Mathematics Study Group in the USA, the work of Papy in Belgium, the Lichnerowicz reforms in France, the West German KMK-Rahmenpläne (cf. Damerow, 1977); and, in England, the work of the Midlands Mathematical Experiment, the Swansea Scheme and that of the early School Mathematics Project.

However, the conjunction of the names of the SMP and MME with those of Dieudonné, Papy and Lichnerowicz should act as a warning that even within this one movement there were many shades of emphasis.

In Belgium and France, 'new math' meant studying the 'affine plane' before the Euclidean plane; it meant an emphasis on algebraic structure and on axiomatics and rigour. In England, and, for example, some Eastern European countries, 'new math' also meant increased emphasis on applications, for example, statistics, probability and programming (iterative processes, flow diagrams); a bias apparent in the meetings held in Oxford, Liverpool and Southampton (Thwaites, 1961) which helped prompt English reforms.

2. *Behaviourist:* the driving force for this movement came from outside mathematics, from educational psychology. Behavioural psychologists, some of whom had recently prepared successful training programmes for US military personnel, wanted to demonstrate how their methods, often used in conjunction with new technological products such as the computer and teaching machines, could improve the success rate of instruction. Such means, it was argued, could help education move from being 'labour-intensive' to being 'capital-intensive'. On the one hand, there was a shortage of mathematics teachers, and general dissatisfaction with the level of numeracy of school leavers; on the other, foundations for a behaviourist approach had already been laid by Thorndike, and mathematics teaching could be (and in some senses was being) readily fragmented into 'tasks'. Mathematics, then, appeared a 'suitable case for treatment'.

It was a movement which was to attract greater attention in the US than in Europe. There, under the influence of psychologists such as Skinner and, more particularly, Gagné and Glaser, various projects were established including, for example, the Greater Cleveland Mathematics Project and Individually Prescribed Instruction. Attempts to introduce Computer Assisted Instruction in mathematics were made in several states. In Europe the principal project was the Swedish IMU Project (Individualized Mathematics Instruction).

The movement was not to have any immediate influence in appeared in programmed form, and attempts were made to introduce teaching machines in one or two LEAs (e.g., Surrey), but these had little appeal. However, the National Council for Educational Technology (as the Council was originally called) showed considerable interest in such methods in the late 1960s: an interest manifested in the 'Resources for Learning' and 'Continuing Mathematics' projects and in a review of CAI in the United States (see, for example, Black, 1969; Howson and Eraut, 1969; Leith, 1969: Taylor, 1971). Later the Council for Educational Technology was to mount a more general investigation into computer-assisted education.

In England, the influence of the behaviourists is most seen, therefore, in secondary form. There is now considerably more attention paid to the enunciation of objectives for teaching units (occasionally these are to be found couched in behaviourist terms) and the work of Bloom and others on evaluation is given considerable attention. It could be said also that the influence of behaviourist thought can be discerned in some of the later workcard schemes to appear, for example, SMP 7-13.

Again, though, there is a big difference between IPI Mathematics with its tightly structured pre-tests, post-tests, etc., on the one hand and SMP 7-13 on the other.

3. *Structuralist:* this movement arose from the study by genetic epistemology theorists, such as Bruner, into the processes of concept formation. Yet, unlike the second movement, its foundations lay not only in psychology but in mathematics itself. For an essential step in this movement is the identification of those structures and processes that are specifically mathematical and by means of which professional mathematicians operate. These 'structures of the discipline' will then, it is believed, prove suitable for promoting learning processes in an optimal way. Children will meet such structures in a variety of embodiments over a long period of time - on each occasion the structure being explored and acquired in greater depth. This is achieved through the device of the 'spiral curriculum'. Moreover, 'discovery learning' will simultaneously serve as a means and an end, for, whilst 'discovering', the exploring student will not only acquire structures, but will behave like a mathematician having full mastery of the structures.

Again, the label 'structuralist' can be attached to widely differing projects. However, the names one most associates with this movement are Bruner and Dienes. (Its origins might, however, be discerned in the quotation from Bagley given in Chapter Two, with its emphasis on 'concepts of method' and 'ideals of procedure'). Dienes, himself, has produced materials in a variety of countries from Papua New Guinea to Canada which exemplify this approach. It is found in Britain in the materials of the Bulmershe Project. However, Dienes' writings (e.g., Dienes and Golding, 1967) exemplify this movement better than do the Bulmershe materials by themselves. Dienes, as the above reference indicates well, initially interpreted Bruner's 'structures' very much in terms of 'algebraic structures' and accordingly he laid considerable emphasis on the provision of suitable 'embodiments' for groups and linear spaces. Later this emphasis appeared to shift. A similar emphasis on algebraic structures is to be seen in much of the work of D.E. Mansfield: in particular, the influential BBC 'Maths Today' series.

Although not so closely tied to Brunerian thought, the South Nottinghamshire Project can also be seen as exemplifying some of the structuralist traits. However, now the emphasis is completely moved from the acquisition of algebraic structures to that of mathematical processes, i.e. the structures stressed are the syntactical (methodological) structures rather than the substantive (conceptual) ones.

4. *Formative:* this is another movement for which the impetus came from outside mathematics. The approach differs particularly from the last in that the determinants of content and method are the structures of personal development rather than those of mathematics. The task of the developer is not to provide 'embodiments' of mathematical structures, but to find out, and match adequately, the content and methods most likely to develop the pupil's cognitive abilities and affective or motivational attitudes or to enhance their development. In this he will almost certainly be influenced by the results of Piagetian research. Indeed, because Piaget's research and that of his followers has mostly been related to the earlier levels of the development of intelligence, and the corresponding theories are well-elaborated for these levels, this approach has been mainly employed by projects working in primary schools.

Other differences between this approach and that of the 'structuralists' are the emphasis on reality and the rejection of models, and the way in which curricular units are intended to initiate learning processes rather than to determine them. (The latter feature means that there is a wide gulf separating followers of this approach from the behaviourists). Yet promoting the autonomous activities of pupils means that the progress of the teaching process will be to some extent uncertain. For that reason, projects cannot aim at producing materials in the form of ready-to-use units. Instead paradigmatic examples of possible procedures are typically offered, together, possibly, with materials which help pupils gain mastery or 'assimilate'.

Projects associated with this approach include, in the USA, the Madison Project and the work of David Hawkins at Mountain View Center, and, in England, the Nuffield Junior Mathematics Project. It is an approach that can also be associated with Caleb Gattegno and, through him, the Association of Teachers of Mathematics.

The Nuffield project, however, also shows the influence of the last of our five 'movements'.

5. *Integrated-environmentalist:* attempts to teach mathematics in a multi-disciplinary context, using the environment as a motivating factor or source of ideas and inspiration. The case for embedding mathematical learning within a wider educational context has been argued on many occasions. Indeed, as we have noted, Dewey and Kilpatrick in the USA argued for the abolition of subject barriers and the use of the 'project' method some sixty years ago. (The same method was also used at that time in Russia in the aftermath of the revolution.) The Spens and Hamilton Fyfe committees considered its possible adoption but rejected it. Spens felt that 'school "subjects" stand for traditions of practical, aesthetic, and intellectual activity, each having its own distinctive individuality ... subjects must be pursued as such' (p.159). Hamilton Fyfe believed that this was a view which had validity when one spoke of the education of the most able, but that it did not necessarily apply 'to the thousands of ordinary children', and quoted the Council for Curriculum Reform (1945) who argued that whatever be the values of the 'subject' carried to full term in university study, they cannot 'be achieved for the child of 16 by simply snipping off a certain length of the "subject" like a piece of tape'. Hamilton Fyfe felt the answer to this last criticism lay in better curriculum design and ensuring that it could no longer be said (as they felt was particularly true of mathematics teaching) that 'the phase of romance is hurried through quickly, the phase of precision is dragged out to cover almost the whole course, and the phase of application, which gives point to the whole business, is not reached at all in many cases' (Hamilton Fyfe Report, p.25).

Yet the attractions of a project-style approach - possibly set alongside or within a more conventional one, remain. The ideas have indeed been resurrected recently by a number of projects. In the USA, the Unified Science and Mathematics in Elementary Schools Project, as its name indicates, emphasizes a multidisciplinary approach in the primary school to the solution of environmental problems. In England, the Mathematics for the Majority Project, and more especially its Continuation Project, have utilised this approach. (The value of such an approach with children of 'average and less than average ability' is argued by Newsom.)

The influence of this approach is also to be detected in some of the suggestions made for teaching applied mathematics in the sixth form (e.g., Brown, 1975) and for teaching modelling in the earlier years of secondary education (e.g., Burkhardt, 1981).

These then are five 'strands'. Almost certainly other educators would select and classify differently. Nevertheless, we now have a finer division within which we can describe and analyse changes.

Chapter Seven
Lessons to be Learned

Evaluation of the changes of the past twenty years is very much subjective. It is impossible to present the reader with a digest of all the reports that have been written. Moreover, the evidence would be so conflicting that it would prove possible, by making a suitable selection, to provide reinforcement and justification for any personal prejudices one might hold. Accordingly, it would seem more objective to have a short chapter which was out-and-out subjective: one which reflected the personal views of the writer, formed as a result of reading, travel and discussion. In this chapter, then, one will find no long quotations and few references. Later chapters will contain references to evaluation programmes, reports of committees, national surveys, etc. However, the reader will more easily pick his way through these, if he possesses a clearer knowledge of the writer's personal views.

We shall consider in turn the five strands identified in the last chapter.

1. THE NEW MATH APPROACH

Perhaps the first lesson to be learned here is that advanced, abstract mathematics will not be made accessible to schoolchildren simply by presenting it with clarity and precision, and 'at a suitable rate'.

The writings of authors such as Papy were exemplary in this respect, yet they ignored problems of motivation and conceptual difficulty. Moreover, such successes as were achieved were normally gained at the expense of 'meaning': vocabulary and formal concepts were transmitted but little which enabled the student to relate these to the rest of his mathematics.

As a result, much of the abstract algebra and geometry (affine geometry approached from an axiomatic standpoint, finite geometries) has failed to retain a foothold in the school.

In England less was attempted in this direction than elsewhere. Nevertheless, for example, attempts explicitly to introduce the notion of a group and the concept of isomorphism with the SMP O-level syllabus were unsuccessful (and the failure resulted in an O-level geometry course which led nowhere in particular). At A-level some parts of group theory remain in the syllabus - but their value is not immediately evident; certainly the form in which group theory is now taught and examined (the two being far from unrelated!) would seem to have little to do with the project's original aims.

In general, however, 'new maths' was obsessed with algebraic structure. The schools were here only following contemporary thought in the universities; in particular, the decline of geometry teaching in the universities set a poor precedent for the schools.

Such 'unifying concepts' as sets and relations have now become established in the syllabi of many countries; matrices in fewer. Yet it would be rash to claim that much had been acheived in the way of 'unification'. The topics have tended to degenerate into the learning of vocabulary and the few techniques needed to deal with a limited range of stock examples. As Godfrey found sixty years ago, symbols have been mistaken for concepts, means for ends. An outstanding example of this is the way in which different number systems have become a subject for study and testing in their own right, and not a means whereby misunderstandings concerning the denary system could be diagnosed and remedied.

This, however, draws attention to the fact that in the majority of countries the 'new maths' movement was directed primarily at students of high ability. Far too frequently, syllabi for the average child were then devised simply by omitting all the difficult mathematics. If we do this, then we are likely to omit the applications (cf. a point stressed in the Hamilton Fyfe report), and often the 'spirit' of the topic.

Was it, however, unreasonable to hope that work on the CAD laws and on binary arithmetic would respectively assist in acquisition of manipulative and numerical techniques? Subjective evidence in England would confirm the empirical findings of the NLSMA (see below) in the USA, that 'comprehension' and 'computation' develop relatively independently (see also Chapter Two, Part A). In England, and elsewhere, there were misjudgements concerning the time needed to be spent on acquiring and practising techniques.

Yet there were successes so far as new content was concerned. In the USA enrolments for 'computer mathematics' and 'calculus' (which it must be recalled was not traditionally taught in US schools) have risen dramatically (Fey, 1978). In England, probability and (elementary) statistics have proved fruitful topics (oddly, they have not 'caught on' in the USA), coordinate geometry and vectors are now introduced much earlier than previously and students now begin university linear algebra courses with a fair knowledge of matrices.

2. THE BEHAVIOURIST APPROACH

There is little evidence, beyond the survival in some countries of individualized 'work card' series, that this approach, based on packages guaranteed to assist in the attainment of specified behavioural objectives, exerts real influence on mathematics teaching.

The totally individualized curricula tended to serve 'subject' ends rather than 'educational' ones. Moreover, once the initial novelty wears off, this kind of work begins to lose its attractions (e.g. in Sweden 92 per cent of pupils wished to continue with IMU after one term, 83 per cent after two, and 72 per cent after three terms. On average pupils also asked for an increase in the amount of time spent on group teaching (Larsson, 1973). Again, the student often learns how to progress through the system quickly rather than pausing to acquire a grasp of the underlying mathematics (see, for example, Erlwanger, 1973). There is also a temptation for success, which can be readily interpreted and measured in terms of correct response rates, to be bought cheaply through an emphasis on technique acquisition and the use of a limited technical vocabulary together with the provision of verbal cues upon which students come to depend.

The '100 per cent' approach also asks much of the teacher. He must resign his traditional position as 'conductor' of the class's learning activities, for one in which he becomes a business manager or administrator. It is not easy to adjust to such a change. Problems of implementation then loom large. (The change will also have to be made to a greater or lesser degree whenever this approach is adopted - even if it is only used in a subsidiary way).

Nevertheless, it would be a pity if reaction to the high pressure techniques of technology salesmen made us overlook a useful aid to pupils' learning and a means of decreasing the load on the teacher. There is ample evidence available of the way in which programmed learning can help achieve limited ends in an extremely efficient way (e.g., Begle, 1979, Bramley, 1979); and in England schools which have used the 'Category 1 - revision' units produced by the Continuing Mathematics Project have commented on their usefulness (Schools Council, 1980).

3. STRUCTURALIST

There has been little empirical evaluation of projects which sought to emphasize the 'structures'of mathematics and accordingly few reports to which one can turn.

It is an approach which is, by its nature, long-term. The basic processes and structures of mathematics must be introduced and gradually reinforced over a period of many years. Moreover, if objective evaluation is to take place then testing procedures of a novel kind have to be designed. Unfortunately, much of the work motivated by this approach has centred on the primary school and teachers in secondary education have failed to recognize and build on the foundations established there. Thus much potentially valuable work introduced by Dienes on attribute blocks, function machines, etc., has never been followed up.

It is noticeable that the two examples quoted refer to concrete materials (or manipulables as the Americans would say) designed to embody very basic mathematical concepts and processes. Unfortunately, the approach would seem to have suffered because in the early days of reform too high-powered, and algebraic a view was taken of 'structures'; thus we find considerable emphasis placed on groups and linear spaces. This had a two-fold effect. First, more basic processes and concepts were ignored. Secondly, many of the embodiments proposed for these algebraic structures were unlikely and bizarre; often they confused rather than clarified matters. Embodiments must be genuine - 'games' must be worth playing.

Yet this is an approach which has much to offer - indeed, of the five described here, it is the only one which would seem to offer a theoretical basis in both mathematics and pedagogy.

4. THE FORMATIVE APPROACH

Evidence would seem to suggest that this approach has had relatively little impact in the majority of classrooms. It has several, not unrelated, difficulties associated with it. The first and most obvious is the problem it presents to the teacher. Unlike the behaviourist approach which claims to be individual, but which can rarely take the problems of individual children into account - indeed which dictates the choice of cognitive style - this approach asks the teacher to treat each pupil as a distinct personality. The teacher must assume added responsibility, yet he/she cannot be given the support provided by the traditional text-book. One cannot supply self-sufficient material which will achieve the desired ends. Yet teachers' guides and and paradigmatic examples have rarely provided sufficient support, even when allied to that given by a teachers' centre.

In direct contrast to the behaviourist approach, this approach would seem to make too many pedagogical demands of teachers.

Other problems arise in connection with the actual mathematics taught. The approach in itself does not provide what one might refer to as didactic mathematical structure. There is a danger that it can degenerate into a variety of unrelated activities. It is also the case, of course, that the theoretical framework which Piaget's writings provide at the primary level does not extend upwards into the secondary school. For such reasons it is not clear what precise guidance can be obtained from this approach at secondary level (cf. Quadling, 1978).

The problems of incorporating 'didactical' structure are even more apparent in our final approach.

5. THE INTEGRATED/ENVIRONMENTALIST APPROACH

This approach, more than any of the others, is bedevilled by the problem of assessment in the later years of schooling. In England we have seen how an attempt to build project work into the external examination taken by those following the 'Mathematics Applicable' course has had to be abandoned. (We note that, in fact the project concerned utilized aspects of this approach instead of adopting it in a whole-hearted manner.) Such problems, of course, do not apply to primary-school children or to those of lower ability in secondary schools. Here, however, the approach raises problems similar to those encountered with the previous one. Much devolves upon the teacher. He/she must find a suitable topic for the pupils to explore. Suggestions can be given in teachers' guides but local conditions and interest make it necessary for teachers to adapt such suggestions to their own needs. The teacher will then have to guide investigations and judge whether to let them continue, expand in scope or to bring them to a close. It may well prove impossible to judge in advance what mathematics will ensue and the teacher will have to be alert to spot and exploit those opportunities which occur. It sets the teacher a difficult problem. In particular, he/she will have to attempt to ensure that there is progression in the mathematics: that a didactic structure exists.

Some attempts have been made to ease the teacher's problems by providing kits based upon environmental or social themes, e.g., the Mathematics for the Majority Continuation Project's *Buildings, Communications, Travel* and *Physical Recreation,* and that produced some years ago at the University of Bath on siting an oil terminal. Yet such kits focus attention on 'the' environment - a general concept, often far removed from the student's knowledge and interest. They cannot, therefore, meet the full challenge of this approach.

One wonders, then, and there is limited research evidence (see Chapter Four, Part A) to support one's doubts, just how successful the 'project' method as a total scheme can be for communicating and teaching mathematics. Indeed, its emphasis on the investigation and solution of specific problems runs counter to that generality which should pervade mathematical thought. It is perhaps relevant to note here that the Tvind schools in Denmark which operate on the 'project' method set aside separate lessons for more conventional mathematics.

Yet the statements to be found in the Spens, Hamilton Fyfe, and Newsom reports concerning the need to make education something more than a selection of academic subjects - the points of which are rarely seen by pupils of average and less than average ability - must be kept in mind. This is an approach which can give purpose to mathematics teaching.

Chapter Eight
Evaluation in Britain

Evalu... on is still in a primitive state and, not surprisingly, the manner in which evaluatory studies have been undertaken has been extremely varied.

> 'Some have been focused on one aspect of a project, others have been comprehensive. Some have generated numbers, others ideas. Some have determined the fate of a project, others have been ignored. Some have been thorough and systematic, others cursory and haphazard' (Howson *et al*, 1981, p.236).

In Britain formal evaluation has not been emphasized so much as elsewhere. All projects have used formative evaluation of some type or other, but there has been little in the way of formal summative evaluation. To the writer's knowledge no Schools Council project was wound up in mid-stream on the grounds that it was getting nowhere - although one publicly-financed project was the subject of a four-man enquiry at one stage of its life.

Yet evaluation has taken place on a variety of levels. Teachers, parents, industrialists and even politicians have all been willing to evaluate the changes on the basis of whatever expertise they possessed. The key questions of evaluation - of whom, by whom and for whom? - have been raised on numerous occasions.

Teachers have evaluated projects by buying or not buying their materials, by continuing or dropping such work. Yet it must not be thought that such an evaluation is necessarily one which correlates with intrinsic worth. On several occasions abroad (e.g., Cundy, 1978), an evaluation study has concluded in favour of an innovation and has spelled out why it is succeeding, only for the project concerned to collapse like a punctured balloon.

Britain has one of the few curriculum development projects, the School Mathematics Project, which enjoys financial independence and so has a continuing life. Here, there is no obvious 'summative' evaluation on display, for the task is seen as never-ending. Instead, we have a type of 'formative' evaluation applied not only to chapters, but also to 'epochs' or 'stages'. Thus, for example, the publication in 1974 of *Manipulative Skills in School Mathematics* can be viewed as a piece of evaluation by members of the SMP who believed, on the evidence as they interpreted it, that errors had been made in the original assumptions concerning the acquisition of techniques and/or the preparation and dissemination of materials. Similar 'informal' evaluation led to the revision of the A-level books and more recently the establishment of a project to produce new materials throughout the 11-16 range.

In a sense, this is the kind of evaluation which led to numerous editions of Durell, Godfrey and Siddons, etc. Now, however, the problems and the aims of the authors concerned are altogether different in scale. It is perhaps this question of scale which has prevented revised and more polished editions appearing as regularly as they did in the past. SMP Books 1-5 have remained unchanged (bar metrication) for upward of fifteen years. The third edition of Godfrey and Siddons' *Geometry* appeared nine years after the first. (Yet, as evidence of the retrenchment that took place, we note that the fourth (1926) edition remained in print for over 40 years.)

Much evaluation has also taken place within the inspectorate; indeed, the publications, *Mathematics* 5-11 (DES, 1979a), *Aspects of Secondary Education in England* (DES, 1979b) and *Mathematics in the Sixth Form* (DES, 1982) can be viewed as evaluative studies.

Yet, there have been attempts of a more formal nature and we attempt below to provide a brief survey of these.

The Schools Council Study, *Evaluation in Curriculum Development: Twelve Case Studies* (1973), would seem an apt starting point. Munro when reviewing the book in the *Journal of Curriculum Studies* was brutal:

> 'Of the twelve studies, only two ... avoid self-validating private rituals and qualify as attempts to assess publicly educational worth' (Munro, 1974, p.175-6).

The two mathematics projects which provided case studies were heavily criticised:

> 'The most worrying example is in the report on the *Sixth Form Mathematics* evaluation where a multiple statement of a curriculum credo is offered as the project's curriculum hypothesis. Furthermore, this hotchpotch of aims, assertions and hunches is couched in such meaningless clichés as to make it quite impossible to test'

and

'The most remarkable statement here is made by the evaluator of *Mathematics for the Majority*. "The major problem in evaluating the project's published materials lay in the existence of an intermediary, the teacher, between the project's material and the children's output". This surprised discovery of the existence of the teacher suggests that the evaluator had come to see his own work as more important that than in the schools themselves' (op. cit. p.177).

In fairness to the projects concerned it must be said that no British project presented its aims in a way that would provide a ready-made frame-work for the evaluator (in fact, only those projects belonging to the behaviourist school can do so - our knowledge of, and techniques for, evaluation are so limited). Moreover, once one (correctly) starts concentrating on the influence of the teacher, then the problems revealed (see Morgan, below) are immense.

Little that positively related to mathematics, then, emerged from that particular Schools Council study. Nevertheless, the *Mathematics for the Majority Project* evaluator did reveal the disturbing statistic that over half of the pupils classified as 'below average' were being taught mathematics by non-specialist teachers (those teaching mathematics part-time, headteachers, etc.) (Schools Council, 1970.)

Several attempts have been made to evaluate the changes that have taken place in the primary school classroom. Perhaps the most extensive (devoted entirely to mathematics) is that by Murray Ward (1979). He explains how

'it soon became clear that we had to go beyond labels like "Nuffield", "Dienes" or "traditional". In the early sixties, a study of mathematics divided schools up in this way. But now most schools employ - quite rightly - a mixture of methods, use more than one text-book and do not rely exclusively on a single form of structural apparatus ...' (p.12).

One notes here the approval given to 'a mixture of methods'. It is, of course, safe; it avoids un-British extremes. But as we have seen earlier, Dienes presents not just a set of activities from which you can take your pick - but a structured, theoretical response to the problem of curriculum construction. What sense does it make to select from a structured scheme - and are teachers mathematically and pedagogically aware of the choices they are making?

Summing up, Ward writes that we are unlikely to see further mathematics reforms of the magnitude of those in the 1960s; that nowadays the difficulty of curriculum development is recognized. The major difficulty is, of course, in changing the teacher's knowledge, outlook and aims.

'For the quality of mathematics teaching depends almost entirely upon the quality of teachers: "The difference in effectiveness lies not in their allegiance to any one method but in: (a) the quality of their relationships with children, (b) their degree of expert knowledge, and (c) their sensitivity in matching their teaching with each child's current learning needs." This extract from the Bullock Report was written with reading in mind; it is equally applicable to mathematics.' (Ward, p.58)

This statement is, indeed, a true one, but it should not be allowed to obscure the fact that the problems in mathematics are greater than those of reading simply because we are not so certain about ends: objectives when teaching 'reading' are well-defined and generally accepted, whereas for mathematics this is not the case.

Several research findings relating to the primary school curriculum are described in J.D. Williams' *Teaching Techniques in Primary Mathematics* (1971). In particular, there are interesting results relating to the use of structural apparatus including one attempt at comparative evaluation by Brownell (1964) which compared the results of teaching 'by the Cuisenaire method, the Dienes method or a conventional method.' Brownell concluded, like so many others, that 'teaching quality is more important than method' (Williams, 1971, p.29). This conclusion is supported by the findings of Biggs (1962, 1967). Biggs remarked also on the manner in which 'analytical methods (e.g., Cuisenaire) are particularly helpful only with boys of high intelligence' - a conclusion supported to some extent by American research (Barszcz, 1974). Williams' book, however, covers much more than curriculum-orientated research.

Hewton (1975) presented an evaluation of the Nuffield project based on sales indicators, interviews with people associated with the project, and statements in books, journals and the press. He showed that the data for adoption led to the usual S-curve in which a small number of early adoptions is followed by a surge and then a decline. However, in the case of Nuffield mathematics the surge came almost immediately, probably reflecting publicity, a felt need for change, the current popularity of innovation, and the lack of competition. Hewton judged the quality of the in-service courses for teachers provided by the specially created centres to be mixed (a finding confirmed by other small-scale surveys of in-service training, e.g., Leopold, 1974; Vallom, 1976). Teachers also felt that other support was inadequate. The guides were found complex and the lack of pupils' materials created a vacuum, largely to be filled by the 'Fletcher' series.

An exercise bearing certain similarities and relating to secondary mathematics was undertaken by Tebbut (1978).

He attempted to find a model to describe the dissemination and adoption of Nuffield Science and SMP materials. He based his workings upon the entries for the SMP O- and A-levels. However, no recognition was taken of schools which used SMP books but entered candidates for the 'modern syllabuses' established by various boards and which closely followed the SMP syllabus. As a result his observations are somewhat suspect.

A more valuable contribution to our understanding of the problems of 'institutionalized' curriculum development is the case study by Hayter on the Continuing Mathematics Project (Schools Council, 1980). This project had more than its fair share of burdens to bear: for example, its 'political' conception leading to over-ambitious goals; rapidly changing staff, two masters (the Council for Educational Technology and the Schools Council); an ill-defined, but wide-ranging, target audience; an emphasis on changing classroom methods (always more difficult to achieve than changes in content); and no coherent policy on publication. In the years between its conception (1968) and its launching, the RDD model on which it was based was already becoming discredited. It is remarkable that something of value should finally emerge from such a troubled project. The story is sympathetically and fairly told by Hayter, a former project member. It is very much a subjective account based upon discussion, documents and personal experience, and it concentrates more on the establishment of the project and the preparation of its materials than on their effectiveness in the classroom. Nevertheless, the account is a model of its kind and a valuable contribution to studies in curriculum development. Other less detailed, but still thought-provoking, project evaluations are to be found in Stenhouse (1980).

Other rewarding studies in evaluation were to come from Scotland. First, the Fife Mathematics Project was the subject of an 'illuminative' evaluation by Crawford (1975). This was conducted via interviews with teachers and pupils and the use of questionnaires.

Unfortunately, it is often the case that people tell the interviewer what they believe will please him, rather than what they really believe. To some extent, then, Crawford's methodology is suspect. Yet his summary (p.133) is probably valid:

> 'the Project seems to have been fairly successful in solving some of the more immediate problems associated with mixed-ability teaching ... Whether the wider educational aims ... have been achieved has not been reliably demonstrated ... The Project clearly makes new demands on the teacher, but there is no sign that this is causing a significant problem'.

Not, one fears, a very definitive conclusion. Indeed, the variety of ways in which schools used the scheme made it difficult to reach significant conclusions on the effect using the project's materials had on the child's acquisition of conventional knowledge.

The actual demands made on teachers, and their reactions to these were more deeply investigated by Judy Morgan (1977) in her study of the SED individualized learning project. She also used talks with teachers and children, but supplemented these with classroom observation. The report highlighted yet another commonly found feature of change: the great variation in teacher reaction and implementation. It is particularly valuable in the way in which it examines the notion of 'self-explanatory' workcards and shows how this influences the actions of teacher and pupil; in the manner in which it demonstrates how individualized learning tends to emphasize speed and the writing down of answers, rather than the learning of mathematics (cf. Erlwanger, 1973, and Smith and Pohland, 1974); and the need for teachers to reframe their ideologies, and to adjust to a new position in the classroom. What the report does not do (intentionally) is to offer recommendations or possible solutions. What it does is to help us understand more clearly why some of the hoped-for reforms of the 1960s failed to materialize.

There have been no comparable studies in England. The Schools Council Curriculum Review, based on Nottingham, in general interpreted 'curriculum' in the narrower sense of 'syllabus' and concerned itself more with mathematical than classroom matters. This is not to deny the value of the work produced. However, it was in only a very limited sense an evaluation.

Several postgraduate theses have been on topics connected with curriculum development, e.g., Preston (1973), which attempted some comparative evaluation in the affective domain. The sample was, however, biased and not very large. Indeed, such work serves more as a peg upon which to hang the question 'Can isolated Ph.D. (and other) theses make a genuine contribution to curriculum research?' One fears that the answer is 'No'. In general, all they will offer are possible methodologies and pilot studies. Time, money and other resources are not available to turn them into anything more valuable. If curriculum research is to be undertaken then other means must be utilized.

What can be achieved by an experienced full-time worker (such as Morgan) was also demonstrated in a different manner by Heard (1978). Many complaints have been made by university engineers of the debilitating effects of modern mathematics on their students. Heard carried out a study in 18 universities and reached the conclusion that: 'There is no evidence that students who have followed one type of

A-level mathematics course perform consistently better or worse at university than students who have followed another type' (p.47). (The benefits, indeed necessity, of having a large sample size is amply demonstrated here for, *en passant*, Heard demolished the tentative conclusions of a pilot study by Cornelius and Marsh, 1977.)

Heard's report also helped to throw light on another issue: the necessity for, and benefit of, studying 'double-subject' mathematics in the sixth form. It will be remembered that one of the early aims of the SMP (Thwaites, 1972) was to discourage what was seen as excessive specialization on the part of A-level students. Indeed, and the causes are not one believes wholly attributable to the SMP, the percentage of A-level students offering the double subject has fallen. What is the effect of this on students and on universities? Heard supplies a partial answer: in their first-year mathematics and engineering examinations, engineering students with double-subject mathematics perform better than those with only single subject. A further piece in the jigsaw was supplied by Cornelius and Irish (1975), who showed (using a small sample) that 'single subject' students following an honours course in mathematics performed significantly worse in their first-year examinations, but later recovered to obtain the same 'mean degree score'.

We have referred to the subjective evaluation of modern mathematics syllabi by university engineers. A more objective type of evaluation intended to confirm/rebut the industrialist's oft-quoted beliefs concerning the innumeracy of the average pupil (whether trained on a 'modern' or 'traditional' diet) was carried out by the IMA (1978). That, for example, approximately half of the 15 and 16 year-old pupils sampled were unable to work out the sale price of a pair of jeans marked at £15 and subject to a 20 per cent reduction is, indeed, alarming. However, it is difficult to decide whether the position has in fact deteriorated and, if so, to what extent. Certainly, the results obtained on the standardized tests to be found in Ballard (1923) and the data supplied by MacIntosh (1979) suggest that the inability of mathematics teachers to impart a high degree of arithmetical competence to the general population has always been a matter for alarm.

The establishment of the Assessment of Performance Unit should, however, mean that in future we have better measures with which to compare performance at different times.

The results of evaluatory studies in the United Kingdom, then, have been somewhat meagre. At best they have increased our knowledge of the problems of curriculum development - in general, they have confirmed the crucial role of the teacher (whilst giving little hint as to exactly what makes a 'good' teacher). Yet the cynic, looking across the Atlantic, might well take refuge in the fact that although little has been learned it has been at relatively low cost.

ERIC

Chapter Nine
Evaluation in the USA

If evaluation has been a cottage industry in the UK, it has been big business in the USA. We can therefore present only a brief indication of what has taken place. When preparing this section the writer has drawn heavily on the NACOME Report (1975), Fey (1979) and the relevant chapter in Howson, Keitel and Kilpatrick (1981).

The 1960s saw an enormous amount of curriculum activity in the USA. When this failed to achieve the promises and hopes of the innovators there were more recriminations than we have experienced in the UK. Kline's *Why Johnny Can't Add* (1973) was a powerful attack on the new math movement by a leading figure in American mathematics - moreover, one who could write in a popular manner. Immediately, mathematics educators rushed to defend their works (e.g., Begle,1974, Wheeler, 1974), but the most considered response was to come from the National Advisory Committee on Mathematical Education of the Conference Board of the Mathematical Sciences (NACOME, 1975). This is an important document which is difficult to summarize. However, there are certain key passages worthy of particular consideration:

> '1) When it has been possible to compare similar classes using traditional and modern mathematics texts, there has been a tendency for the traditional classes to perform better in computation while the modern classes perform better in comprehension.
>
> 2) There appears to have been a decline in basic scholastic skills since 1960. Mathematics achievement has shared in this decline. Recalling our observation ... that the reform movement was not widely implemented in the classroom, there is still some evidence that secondary classes using a modern program have tended to resist the general decline in achievement.' (pp.118-119).

The problem of teacher preparedness was also raised:

> 'Despite their general willingness to try new curricula and teaching methods, elementary (primary) teachers are seldom mathematics specialists and few in-service training

programs prepared them to exploit fully the letter and spirit of new curriculum materials. The reconstruction of K-6 mathematics was also dependent on information from psychological studies, including many still unanswered questions about growth of mathematical abilities in young children. This barrier to effective curriculum development was compounded by the limited role of experienced elementary school teachers in development terms' (p.11).

One response to the apparent decline in mathematical standards was the attempt to establish 'minimal mathematical skill lists - various state departments of education, an NCTM Committee and many individuals have generated basic competence recommendations' (p.53).

NACOME did not propose its own list but asked those proponents of 'minimal competency tests' to consider potentially serious drawbacks.

'First, the objectives intended as a minimum ... can all too easily become a ceiling also. Particularly when the objectives are tied to an accountability assessment ... Second, focus on minimal skill goals can inappropriately constrain planning for instruction by suggesting that skills must be acquired in a rigid sequence of mastery level steps ... Third, in a cumulatively structured subject like mathematics, it seems highly unlikely that fundamental concepts and skills will be passed over by instruction reaching for higher goals ...

If the task of identifying minimal mathematical competencies leads to a careful analysis of the fundamental concepts and methods that have characterized the discipline throughout its history, it can provide useful insight for the curriculum development and teaching. If, as we fear, the search is for a list of easily taught and easily measured skills it will not be productive' (p.53).

Despite these warnings the minimal competencies bandwagon would seem to have acquired still more passengers in the years following the publication of the NACOME Report.

Another interesting, post-NACOME, general survey of US mathematics is provided by Fey (1979) who provides a summary of three national surveys: a comprehensive critical review of the literature on curriculum, instruction, evaluation and teacher education from 1955-1975 (Suydam and Osborne, 1977); a collection of surveys directed at teachers, administrators, parents, and students in grades K-12 (Weiss, 1978); and a collection of case studies in selected schools and districts (Stake and Easley, 1978).

Some guide to the popularity and esteem is given by secondary school enrolment numbers. There has been a rise in the number enrolling for geometry and a rapid growth in computer mathematics and calculus enrolments. Yet in the five years 1972-1977 there was a fall in the numbers taking advanced algebra and advanced senior mathematics: 'there is a large body of less able or less ambitious students who are electing to stop their high school mathematics preparation after [10th grade] geometry' (Fey, p.492).

Surveys undertaken in 1976-1977 confirmed the NACOME view that the federally sponsored innovative curricula in mathematics had had disappointingly little long-term influence on school practice. In particular, teaching styles have remained largely unaltered.

> 'Certainly the mathematics teaching these observers saw embodied none of the spirit of inquiry, laboratory exploration, or individualization that has been so strongly urged by a variety of experts. But as Welch goes on to indicate, "Although it seems boring to me, students and teachers seemed comfortable with it. Apparently it fulfils student expectations and provides the teachers opportunity for closure"' (Stake and Easley, Chapter 5, p.6).

This is perhaps too easy an explanation for, in general, the surveys disclosed that

> 'students find study of mathematics boring and teachers find motivation ... one of the most difficult problems they face. When teachers were asked what aspects of their jobs they most needed help with, at all levels they mentioned learning new teaching methods and implementing discovery/ inquiry approaches' (Fey, pp.495-6).

Again, all the reports stressed the crucial role of the teacher, but 'there is no demonstrably superior way to identify the knowledge, experiences, or personal traits of people who will be consistently effective teachers' (Fey, p.496).

A particular feature of American education to which many teachers (over 90 per cent of junior high-school teachers interviewed in one case study) objected was the way in which students could be 'promoted without knowing basic mathematics'. (The old tradition of 'repeating' years still operates in many Eastern European countries - see, for example, Swetz, 1978.)

> 'When really pressed to find virtue in the study of mathematics, most teachers responded with variations on the familiar mental discipline argument ... The students seem to sense this same underlying objective in studying mathematics, though they express somewhat more cynical

regard for the goal' (Fey, p.498).

Teachers, it was found, were less than happy with their present lot:

> 'I always thought that the main goal of education was teaching kids, now I find out that the main goal is management' (Stake and Easley, Chapter 2, p.9).

The author of one case-study was led to characterize the condition of mathematics teachers as a mix of:

> 'flatness, a lack of vitality, a seeming lack of interest in the curriculum by both the teacher and the children, a lack of creativity and curricular risk taking, a negativism toward the children - they're spoiled, they don't care, they don't try - and sometimes a negativism toward colleagues, administrators, and college and university training programs (often decades ago)' (Stake and Easley, Chapter 3, p.84).

It was felt that the reason for this was often 'the isolation of a teacher's work in the classroom'. Those teachers

> 'who had maintained contact ... with scientific and professional communities beyond the school, had 'kept open a window on the larger world of ideas'. But ... 'Most teachers have only a mirror that reflects the values and ideas already dominant in the ... schools''(Fey, p.499)

Summing up, Fey writes

> 'The most discouraging feature of the three NSF studies is the consistent pattern of great differences between apparent reality of mathematics education in most schools and the recommendations or practices of many prominent teachers, supervisors, and professional organizations. For instance, it appears that a large majority of elementary teachers believe that their sole responsibility in mathematics teaching is to develop student facility in arithmetic computation ... Secondary school mathematics teachers find it difficult to motivate their students or to induce lasting learning. They express interest in learning new teaching ideas, yet seem basically satisfied with their current methods and objectives' (p.504).

Finally, in this section we look briefly at the way in which evaluation studies have thrown light on the five 'strands' of curriculum development identified in Chapter 6.

1. <u>THE NEW MATHS</u>

The largest study here (and anywhere else for that matter) was the National Longitudinal Study of Mathematical Ability, a multi-million dollar essay in evaluation. NLSMA used a sample of 110,000 pupils from 1,500 schools in 40 states. However, this population was neither random nor representative of the US system as a whole (in retrospect, an astonishing fact). Its methodology leaned heavily on Bloom's taxonomy (see the chapter by J.M. Wilson in Bloom *et al.*, 1971). Considerable data was collected and information generated - some of which, e.g., on the relative performance of modern and traditional students on manipulation and comprehension, was used by the NACOME committee. NLSMA also showed that modern text-books produced widely different effects on student learning although investigators were usually at a loss to explain why. The effect of text-book choice in general diminished over the years, indicating perhaps that teachers at lower levels adhere more to the class text.

Detailed summaries of the NLSMA findings are to be found in Osborne (1975) where one can note that several reviewers were bothered by the complexity of the analysis and the fallibility of the data.

One lesson to be learned from such a study is, to quote a participant, 'that the biggest temptation in a longitudinal study is to spend so much of one's resources garnering and organizing the data that one has no energy left to analyse it'. (Howson *et al.*, 1981, p.194).

2. THE BEHAVIOURIST APPROACH

Consistent formative evaluation is a feature of the behaviourist approach - are units successful in achieving the relevant behavioural objectives, are students proceeding through units at a 'reasonable' rate? Lindvall and Cox (1970) describe just such an evaluation of the IPI Mathematics scheme. It is an inward-looking evaluation study aimed at seeing if limited, self-set goals are being achieved. More critical analyses of IPI were to come from outsiders. Oettinger (1969) observed that 'The children are happy and eager, but what are they learning? They are learning about that valuable but restricted range of human knowledge and attitudes that can be mechanically expressed and measured. Without other forms of education, they may grow up under the dangerous illusion that there always exists a correct answer to every question' (p.148). The most devastating criticism of IPI came, however, from Erlwanger (1973, 1974). He found

> 'its behaviouristic approach to mathematics, its mode of instruction, its form of individualisation, and its evaluation and diagnostic program inhibited the development of the children's intuitive ideas, (and) thereby encouraged the development of their misconceptions.'

His findings on 'teacher-effect' confirmed those of Morgan (see above). Other confirmation of Morgan's findings concerning the pressures quickly to supply an answer, and to complete more units than one's classmates, was supplied by Smith and Pohland (1974) who investigated the use of CAI (computer-assisted instruction) in schools.

3. STRUCTURALIST

The lack of incentive and adequate test methods has inhibited evaluatory studies in this area. The problems of evaluating 'discovery learning' were discussed by Shulman and Keisler (1966). (More recent work has been referred to in Part A.) However, the Comprehensive School Mathematics Project (with a pause for breath after Comprehensive and *not* School) was the subject of an evaluation carried out by its funding agency the NSF and directed by M. Herbert. First, five members of the Mathematical Association of America were asked to evaluate the soundness and relevance of the mathematical content of the project's materials. Then classroom observers attempted to assess, by interviews and observation, how well the programme was being implemented in the classroom. Costs were considered as was the effectiveness of the in-service training provision. It was found that pupils' learning was generally adequate, and was excellent in some cases. Pupils in CSMP classes usually outperformed pupils in comparison classes on achievement tests of various kinds. Criticism was, however, levelled at some embodiments - in particular the Papy minicomputer and its value when used over a wide range of ability. (See Herbert, 1974.)

4. THE FORMATIVE APPROACH

Again evaluation has proved difficult and there have been few formal attempts to evaluate this approach. However, studies carried out in California (Dilworth and Warren, 1973) indicated that in-service training schemes provided in connection with the Miller Mathematics Improvement Program yielded relatively consistent and reliable gains in the achievement of the pupils of participating teachers. (This is the type of result one would naturally expect to record, but regrettably not one which has frequently been obtained in research studies.)

5. THE INTEGRATED TEACHING APPROACH

An evaluation of the USMES project was undertaken by Mary Shann and others (Shann *et al.*, 1975). In particular, NSF, the funding agency, was anxious to see if USMES was succeeding in increasing pupils' abilities to solve real problems. Attempts by Shann to broaden the scope of the enquiry met with limited success. However, interviews were conducted with teachers, pupils and administrators and studies were made of forty USMES classes and forty matched control classes. These studies showed no statistically significant differences in mean achievement between the two sets of classes on tests of reading, comprehension, mathematics, science, and social science. Following the USMES course, then, had no deleterious effects. However, attempts to test whether or not the USMES groups were better at solving real problems also showed no significant differences in achievement. What Shann did reveal was the difficulty of promoting interdisciplinary work in primary schools. Teachers wondered if USMES was teaching problem-solving or 'gimmicks', and were able to find little science in the units. The failure of teachers actually to implement USMES work in schools was often frightening. Moreover, the report clearly revealed, in a way unique in the present writer's experience, how the 'official' view held by teachers and administrators about a project differed from what they were willing to express in confidence - but 'Don't say anything bad about me back in Boston. Tell EDC what a good job I am doing so I can go back to Boston again next summer' (Shann, 1975, p.252).

Chapter Ten
The Management of Curriculum Development

The general problem of how curriculum development can best be managed has been considered by many authors (e.g., Owen, 1973; Becher and Maclure, 1978). The most comprehensive studies have however come from IMTEC, originally sponsored by CERI/OECD (when the acronym denoted International Management Training for Educational Change) and now independent (and retitled The International Movement Towards Educational Change). IMTEC prepared an impressive series of publications describing various case studies of innovation from the central to the school level (OECD, 1973). Few of these related specifically to mathematics, but later Dalin, the director of IMTEC (Dalin, 1978), was to synthesize the lessons of these case studies in his *Limits to Educational Change*. It is impossible to précis such a spare and valuable work. However, it has much to say on the processes of change and on the strategies used to initiate and disseminate innovation. It charts the downfall of the Research - Development - Dissemination model of educational change; that import from industry which seemed so appealing in the 1960s. We see how the climate has changed in favour of 'periphery' located projects. Thus, the old model of materials being produced centrally by specialists and then disseminated into the schools on the periphery (Nuffield Mathematics Project, Mathematics for the Majority Project, Schools Council Sixth Form Project) is gradually being abandoned. In its place we have locally-based projects with greater teacher participation (Mathematics for the Majority Continuation Project, Kent Project, SMILE).

Certainly, such a move eases problems of dissemination. Yet merely bringing teachers together and inviting them to write their own materials is likely to be as unrewarding a strategy for curriculum development as is the RDD model. It is likely to be more expensive than the latter; moreover, unless considerable professional advice is available the products of 'local' curriculum development can be of a low mathematical calibre. (The inability to understand the work aims of others is a poor qualification for preparing one's own course.) Yet 'its aims and potential are greater, for it seeks not only to change the curriculum, but the teacher also' (Howson, 1979, p.147).

It would seem, however, unreasonable to expect a small group of teachers to combine active learning with the production of a complete school course. Thus projects using this mode have tended to produce 'modular' courses with each of a number of small groups contributing units. There are still dissemination problems, therefore, as teachers are asked to absorb units produced by others. Nevertheless, involvement in the preparation of materials can bring that self-confidence and knowledge that facilitates the acceptance and full appreciation of the work of others.

Yet

> 'Organizing curriculum development in this mode - attractive though it may seem - does not allow us to sidestep ... crucial questions. ... We must still ask "Who initiates such developments?", "How are the writing groups formed?", "Who is to determine the subject matter and aims?", "Who is to determine the style of the materials?", "What resources, financial and other, are required and how are they to be supplied?"' (Howson, 1979, p.147).

General problems of dissemination have been considered in reports such as that by Ruddock and Kelly (1976). The difficulty in writing such 'comparative' reports is that it is all too easy for them to degenerate into an account of differing practices - mainly dependent upon local or national conditions. Finding a way through the mass of data then becomes difficult.

In general, however, 'high-level' institutional practice, e.g., whether the central curriculum development agency takes the form of the Schools Council, the National Council for Innovation in Education (Norway), the National Board of Education (Sweden) etc, is very much a national matter, dependent upon politics and tradition. Moreover, one suspects that it makes very little difference to the success or otherwise of curriculum development, for that depends much more on how the work within actual projects is carried out - and this presents managerial problems of a different form. Certainly, in England one has noted the difficulties of recruiting directors who have the many qualities - mathematical, pedagogical, administrative and personal - required to carry out the job effectively; of recruiting staff on a temporary basis who have proved (or even unproved) writing capabilities (some projects have been plagued by staff changes and early 'retirements' to obtain permanent posts); of appointing consultative or advisory committees which exert any control whatsoever; and of exercising 'quality control' on the project's output.

Chapter Eleven
Centres and Networks for Curriculum Development in Mathematics

We have already mentioned the network of teachers' centres established in Britain. Other countries have also developed such centres on roughly the same lines. However, further types of centre and kinds of networks have been established and these are of some interest.

In France, for example, a chain of Institutes for Research in the Teaching of Mathematics (IREMs) have been established. These are located within universities and, some twenty-five in number, are dispersed throughout the country. They had

> 'a four-fold aim: first ... to study all the factors which affect mathematics teaching, with a view to improvement; to ensure the in-service training of teachers; to contribute to their initial training; and finally to gather together and disseminate the literature generated by earlier researches' (Revuz, 1978).

Initially, the remit of the IREMs embraced only secondary mathematics but later their interests spilled over into primary mathematics. The work of the IREMs has been of considerable interest and of great variety - for the various Institutes have greatly reflected the interests and personalities of their directors. Men such as Brousseau, Glaeser and Glaymann chose to direct their attention to different facets of mathematics education. (Confirming again, if confirmation is still needed, that centralized systems are not as uniform as those in non-centralized systems are often led to believe.) A description of the work of the Lyons IREM is given in Bell (1975).

Like most of the initiatives we describe in this sector, the IREMs have for some years had to fight for survival. Curriculum development as a political cause is somewhat passé - excessive expectations were not fulfilled. The description which Revuz offers of their work must be seen then as an attempt to argue internationally those lessons which he would like to be taken to heart nationally.

Another network, that of the research and development centres and regional educational laboratories established in the USA, has already suffered from major contraction. Twenty-one R and D centres and 20 laboratories were set up in the mid-sixties on a federal basis. These R and D centres, which were based on universities, were intended to bring together scholars to develop programmes dealing with a common area of concern; higher education, teaching, etc. In the event, the successful centres proved to be those which developed some system for facilitating teaching or learning. Examples include the IPI program of the Learning Research and Development Center at the University of Pittsburgh, and Individually Guided Education, a product of the Wisconsin Research and Development Center.

The regional educational laboratories have development rather than research as their prime aim. Each laboratory is a non-profit making body with its own directors, staff and other sources of income. Laboratories have concentrated on developing materials and programmes that are based on work done by the centres and by other researchers. Thus, for example, Research for Better Schools Inc., the regional laboratory in Philadelphia, developed, expanded and disseminated the IPI programme begun by the Pittsburgh centre. (See Howson, *et al.*, 1981, pp.73-5.)

Both centres and laboratories were, however, to suffer as a result of decreased federal spending and increased criticisms of the outcomes of curriculum research and development. As a result, by 1977 their numbers had dwindled to nine and eight respectively.

An outstanding centre which lost its fight for existence was the Dutch Institute for the Development of Mathematics Instruction (IOWO). (Here, it must be noted, the political question at issue is whether a mathematics institute should exist independent of the more general national curriculum centre.) IOWO was established in 1971 under the directorship of Hans Freudenthal and during its lifetime undertook both research and development work (e.g. 'the two projects Wiskobas and Wiskivon directed at primary and secondary education respectively). It sought to obtain maximum participation by teachers in its work (an aim aided by the fact that any teacher in Holland could reach the centre by car in under two hours), and engaged in a considerable amount of in-service work. (See *Educational Studies in Mathematics*, 7, 3, 1976 - an issue devoted entirely to the work of IOWO.)

It was then larger and more nationally and developmentally orientated than the two Shell Centres in Britain.

Yet another centre to be created in the 1970s was the Institut für Didaktik der Mathematik (IDM), the Federal German centre at Bielefeld. IDM is the largest mathematics education centre in the world and bears some resemblance to the US R and D centres in that much of its work is undertaken at a research level. It has provided a European focus for mathematics education research, it offers excellent library facilities - and has been responsible for a vast output of reports, bibliographies, etc. (see, for example, IDM 1980). The British are likely to see it as being too theoretically orientated and insufficiently related to the classroom. Yet there is no doubt that it has helped train a new generation of mathematics educators and this may well prove to be its greatest contribution to mathematical education, provided, of course, that those so trained establish and develop that classroom contact which at the moment might seem to be lacking.

There are then several models (and we have noted only a selection) which can be used to encourage the growth of research and developmental activities in mathematical education.

Chapter Twelve
Common Problems

In this section we provide a brief indication of three problems which are shared with countries elsewhere. We do not quote research findings but simply allude to their existence.

(i) *The problems of undifferentiated curricula:* The comprehensive school movement began in Europe in Sweden, and it was experience there which was much quoted in the 1960s by the proponents of comprehensive education in Britain (see, for example, Pedley, 1969). It is then of interest to see that doubts are now beginning to be expressed in Sweden on the feasibility of undifferentiated curricula:

> 'Perhaps the idea that the school can give all pupils the same education - that is the principle of undifferentiated classes - is not well founded. A more feasible alternative would be to give every pupil the education he needs and can assimilate' (Unenge, in Swetz, 1978, p.250).

(Who decides on what a pupil 'needs' is not mentioned by Unenge.)

The basic quest for egalitarianism has also led Eastern European states to offer a single mathematics course for all - moreover, one which resembles very much the old gymnasium/grammar school course. Not surprisingly, therefore, the problem has been raised of what happens to those pupils who cannot cope. We have already referred to the custom of repeating classes (a fate which according to Castles and Wüstenberg, 1979, p.94, is more likely to befall the children of manual than of mental workers). There have also been criticisms from such as Margot Honecker, Minister of Education in the German Democratic Republic, that

> 'We must not overlook the fact that many pupils have not acquired satisfactorily the elementary mathematical understandings and skills ... The ability to solve word problems has not been developed sufficiently. These deficiencies begin at the lower level and continue through 10th grade' (quoted in Swetz, 1978, p.107-108).

(ii) *Problems caused by the restructuring of school systems:* The problems of comprehensivization have already been mentioned (see also, for example, Viet, 1976, who looks at the effects of the introduction of an 'orientation' year in some of the North German comprehensive schools).

However, the introduction of middle schools and sixth-form colleges has brought problems similar to those encountered elsewhere.

> 'A typical example in France is the creation of the *Collèges d'Enseignement Secondaire*, for children in the age range 10+ to 14+. Because of their decentralized establishment, as their numbers increased they undoubtedly became a force for the democratization of education: but the other side of the coin is that, by the division which their very introduction has helped to bring into being at the heart of secondary education, two distinct worlds have been created, with teachers who work only at this lower level or at sixth-form level having different viewpoints and narrower horizons. And for the pupils the change of establishment as they pass from one level to the other has become yet one more barrier to cross' (Revuz, 1978, p.176).

This problem of the narrowing of teachers' horizons is an important one and must be given particular consideration in the light of the research reported in Chapter Nine.

(iii) *The education of those of high mathematical ability:* Two noted features of East European education are the olympiads and the special mathematical schools.

The former is, indeed, now a feature of UK life. Several articles have been written about the development of the olympiads (see, for example, the ICMI Report (1969) which contains a large bibliography that could now be considerably extended). The ICMI report also contains comments on the pros and cons of olympiads. However, there is little in the way of research findings to indicate how many students have been attracted to mathematics (perhaps misguidedly) through their success in olympiads, or how many have been deterred by their lack of success. Are the tests representative of what we wish to encourage in mathematics, or do they present a very restricted view of mathematical activity? Are we truly selecting advanced mathematical talent, or (as would appear from a glance at the list of schools represented in the British olympiad teams) merely recognizing outstanding schoolteachers and well-run/privileged school departments? Does the total domination of the male sex reflect differences in ability or in opportunity and expectation? Unfortunately, research effort has not been directed at answering such questions.[1]

The mathematical schools have been well documented (see, for example, Vogeli, 1968; Owen and Watson, 1974). Again, the accounts are descriptive and there would appear to be a lack of research on the sociological and mathematical effects of such schools. They nowadays do not lack critics in their own countries. Thus Kapitsa, the Soviet physicist, is quoted as writing that

> 'schools set up for chosen gifted youth in the areas of mathematics, physics, chemistry and biology are proving actually to be harmful ... If a talented child is taken out of a school, this is like bleeding the institution, and has a marked effect on the level of the whole school ...' (Swetz, 1978, p.82).

Ehrenfeucht,writing from Poland, raises this and other objections:

> '1) From a pedagogical point of view it is not good to select a 15 year old student as a 'future genius'.
>
> 2) If it happens that a student just cannot cope with difficulties in such a class, it is a painful experience for him.
>
> 3) It is harmful to specialize too early and deprive a pupil of full education in the humanities.
>
> 4) It is not good to deprive ordinary schools by taking the most gifted away.
>
> 5) The teaching of mathematics in such classes is too abstract, and is not related to applications.' (Ehrenfeucht, 1978, p.288).

Again, then, special schools raise problems for those selected and those who fail to be chosen. There are, however, no relevant research findings known to the writer.

These then are but three problems shared with other countries: further examples would not be difficult to supply.

1. A recent investigation by Margaret Hayman has shown that about 60 per of those past members of the IMO teams who responded to her enquiry had obtained 1st class honours degrees at Oxbridge and that more than half took some higher degree. Of the 31 in employment, 11 are in academic posts and 11 in central or local government. No girl has ever represented Britain in the IMO.

Chapter Thirteen
The Teacher's Role in Curriculum Development

In this survey of curriculum development research and practice it is the teacher who has almost always played the principal role in determining success or failure. His role is central to curriculum development: but what exactly is that role, and how is it viewed in different countries?

In England we have what Maclure (1968) refers to as the 'myth' of the autonomy of the teacher:

> 'This is a myth in the sense that it expresses great truths in a form which corresponds more to an idea than to reality. The less factually correct it may be, the more important it is to retain and embellish the myth' (p.10).

Certainly, the view of 'the teacher as master of his fate and pupils' curriculum' is a myth - for any teacher is hedged about by constraints: examinations, headteacher and parents being but a selection. Yet, the teacher or, more properly, teachers exercise considerably more power in England than they would appear to do in almost any other country. If we consider the 'crucial' questions relating to curriculum development which are set out in Chapter Ten, then we find that in England it has almost always been teachers who have initiated developments, formed the writing groups, determined the style of the materials and, most significant of all, been responsible for determining subject matter and aims. (The contrasting bases, methods and achievements of non-teacher initiated - but still largely teacher-dominated - projects, Nuffield Mathematics, Continuing Mathematics Project, Schools Council Critical Review, and of teacher-initiated schemes, School Mathematics Project, Kent Project, Schools Council Sixth Form Project deserve detailed consideration.)

The effects, benefits and disadvantages of such teacher domination are matters for discussion. What is not, is the fact that in reality very few teachers indeed were concerned in the decision-making processes. Thus, for example, although SMP was 'teacher dominated', the number of teachers actively involved in writing the many texts and planning the syllabi cannot have exceeded 200: yet the materials have been used by many thousands of teachers. Actions, then, have resulted from the efforts of teachers - but these were by no means typical teachers. That the results of their actions often failed to be accepted by the general body of teachers springs from this distinction.

That the few have effectively directed progress is nothing new in our educational history. In the early 1900s, the reforms were effectively led by a score or so of teachers at most. Yet at that time these leaders were dispersed through what was still a very small secondary system. One suspects, too, that there was less diversity to be observed regarding teachers' qualifications, aims and attitudes. Yet what was not stressed then was the role of the 'teacher as curriculum developer'. It was, indeed, implicitly accepted that, subject to the constraints exercised by university and other examinations, the teacher had this responsibility. As we have seen, the way in which teachers responded to changes in the early 1900s was not dissimilar to the reactions observed in later years.

However, particularly in the secondary sector, there have been differences in role caused by changes in the styles of materials being offered.

The traditional arithmetic or algebra text-books usually consisted of a collection of summaries followed by hosts of carefully graded exercises. They provided a framework for the teacher, but left it to him to determine how to introduce and motivate the topic. He was assumed (perhaps incorrectly) to be capable of doing this and in that sense he was a 'developer of the curriculum'. The reformers of the 1960s, however, changed that text-book style for a variety of reasons: the books were often intended to be read by students; they frequently attempted to inform both pupil and teacher; and they attempted to influence, and usually to change, the teacher's style of teaching. Most teachers were now more circumscribed - if not by the text-books themselves, by that lack of knowledge and self-confidence that would allow them to set the books temporarily aside. Even the jokes now came with the materials.

The weaker teachers, of course, relied most upon the text-books and with the same kind of result that Palmer had observed in 1912.

The teacher in the primary school who wished to follow Nuffield had no such text-book to fall back on (until the opportune appearance of the Fletcher series). She had truly to develop a mathematical curriculum - and, simultaneously, one in several other subjects. It was an impossible burden to place on any teacher. A more limited, and possibly more profitable, involvement in curriculum development has recently been offered to many groups of teachers through participation in the drawing up of 'county' and other guidelines for primary mathematics teaching.

Completely new opportunities were offered to teachers (and/or pressures were placed upon them) by the institution of CSE and, in particular, of Mode 2 and Mode 3 examinations. Some teachers took immediate advantage of the new possibilities (see, for example, Fielker, Last (1968)). These modes have, however, tended to be used for less ambitious, low-grade work. For example, we find that of the 47,500 candidates who in 1979 entered for the CSE mathematics examinations administered by the Southern Regional Examinations Board, 11,500 entered for Mode 2 examinations and 5,000 for Mode 3. However, the majority of Mode 2 candidates attempted the limited grade 'Arithmetic' and 'Money Management' papers made available on a 'group' basis. Again a high proportion of the Mode 3 entries were for limited grade syllabi.

Yet does it still make sense to speak of 'the teacher as a curriculum developer' or 'the teacher as an educational researcher'? As we have observed, there are certain tasks which teachers cannot fairly be asked to undertake (without enormous changes in the time they are allowed for preparation and in-service education). Yet, what are the alternatives?

In the USA, the teacher is seen as a consumer of curriculum materials which are carefully constructed away from the classroom. In such a climate the production of would-be 'teacher-free' packages is natural, as, indeed, is the emphasis placed on the behaviourist approach in general. We have seen, however, the limitations - practical and educational - of such an approach.

These lead us inescapably to the view that the teacher must play a part in determining the curriculum of his class - in making those minor adjustments that are necessary to cope with individual personalities, with local interests and conditions, etc. The teacher who says 'tell me what to teach and then I'll do it' is the one who can be safely and economically exchanged for minicomputers plus software.

Yet making such adjustments is not to be confused with preparing a whole course from scratch. If one has such large-scale developments in view, then it is presumptuous to ask for the 'teacher as a curriculum developer', and problems have probably resulted as a result of such calls. Surely, what must be regarded as our goal is that teachers should be prepared (in both senses - ready and equipped) to exercise choice and to make decisions from a position of knowledge and self-confidence. Only then can curriculum development proceed in a non-catastrophic (in a technical sense) manner.

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